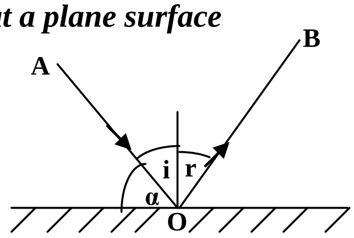
GEOMETRICAL OPTICALS

Reflection of light

Reflection at a plane surface



Ray AO is the incident ray, OB is the reflected ray, ON is the normal line.

α is the glancing angle.

i is angle of incidence

r is angle of reflection.

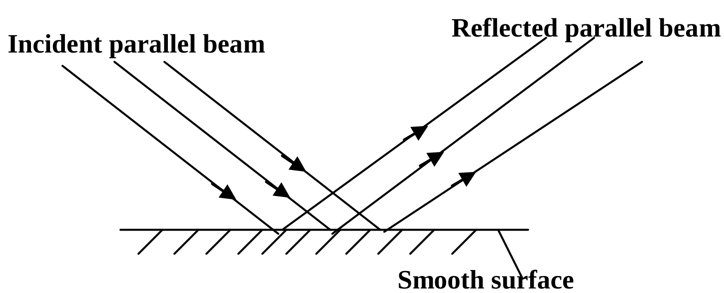
Laws of reflection

1. The incident ray, the reflected ray and the normal at the point of incidence all lie on the same plane.
2. the angle of incidence is equal to the angle of reflection ie angle ***i*** = angle ***r***

Types of reflection

1. ***Regular reflection***

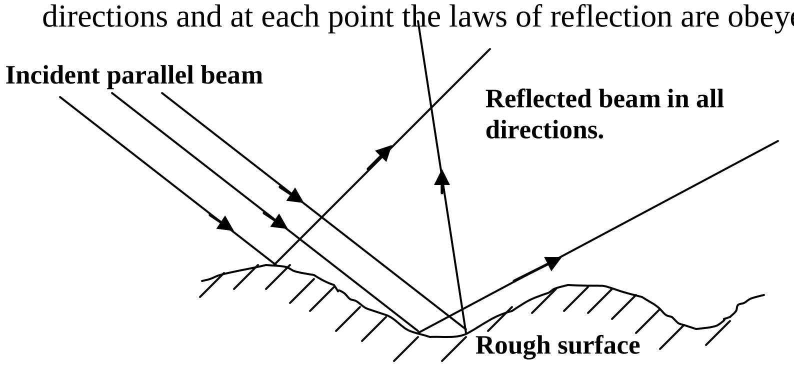
If a parallel beam of light falls on a plane mirror with a *smooth surface*, it is reflected as a parallel beam



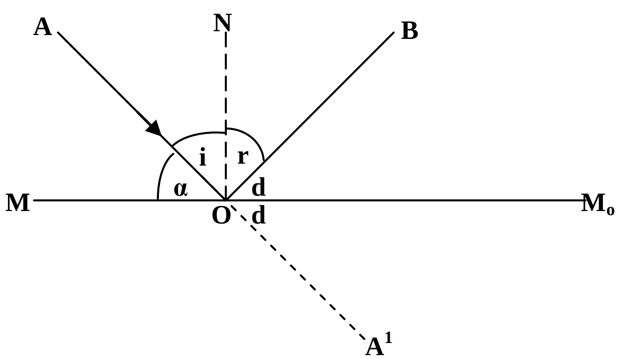
Light is said to have undergone a regular reflection

2 *Irregular reflection or diffuse reflection.*

If a parallel beam of light is incident on a rough surface, light will be reflected in all directions and at each point the laws of reflection are obeyed.



Deviation of light at a plane surface



$\mathbf{MM_o}$ is the mirror surface

Angle BOM_o = 90-r

Angle AOM = α = 90-i

But i = r (law of reflection)

Angle BOM_o = angle AOM= α

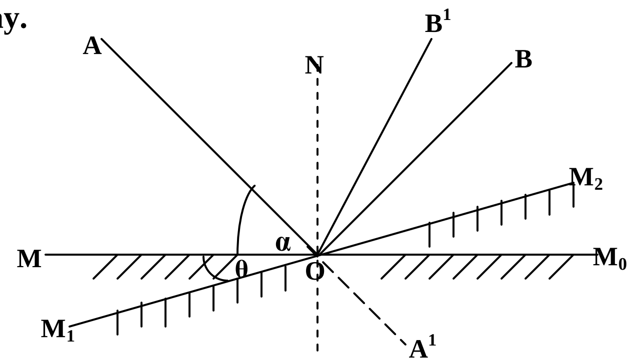
Deviation = angle $A'OM_o + angle BOM_o$

But angle A^1OM_o = α (vertically opposite angles)

Deviation angle = 2α

Angle of deviation produced at plane surface is equal to twice the glancing angle.

Relationship between the angle of rotation of a mirror and angle of rotation of the reflection ray.



MMo is the initial position of the mirror. Keeping the direction of the incident ray constant, the mirror is rotated about O through an angle ϴ to a position M_1M_2

OB is the reflected ray in position MMo of the mirror and OB^1 is the reflected ray in position M_1M_2 of the mirror.

Deviation produced by mirror in position, MMo $= \angle BOA^1 = 2\alpha$

When mirror is rotated to position M_1M2, and the glancing angle = $(\alpha + \theta)$

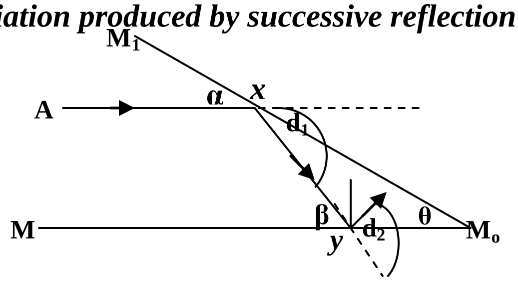
Deviation produced by mirror in position M_1M_2, the glancing angle = $\angle B^1OA^1 = 2(\alpha + \theta)$

Angle of rotation of reflected ray $= \angle B^1OB$

But $\angle B^1OB = \angle B^1OA^1 - \angle BOA^1 = 2(\theta + \alpha) - 2\alpha = 2\theta$

Therefore the reflected ray is rotated through an angle which is twice the angle of rotation of the mirror.

Deviation produced by successive reflection of two inclined mirror



Consider two mirrors M_1M_o and MM_o inclined at angle ϴ. Consider a ray AX incident to the mirror MM_o at a glancing angle of α.

Deviation produced at X, $d_1 = 2\alpha$ (Clockwise)

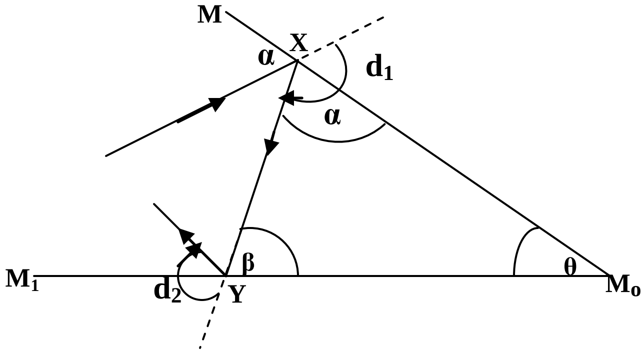
Deviation produced at $y = d_2 = 2$ β(anticlockwise)

Net deviation $= d_2 - d_1 = 2\beta - 2\alpha = 2(\beta - \alpha)$

But $\beta = \alpha + \theta$ (sum of two interior angles is equal to the opposite exterior angle)

$\theta = \beta - \alpha$

$\therefore$ Net deviation $= 2(\beta - \alpha) = 2\theta$ (anticlockwise)



Deviation at x, $d_1 = 2\alpha$ (clockwise)

Deviation at y, $d_2 = 2\beta$ (clockwise)

Net deviation = $2\alpha + 2\beta = 2(\alpha + \beta)$ Clockwise

But using triangle XYM_0,

$\alpha + \beta + \theta = 180^0$

$\theta = 180^0 - (\alpha + \beta)$

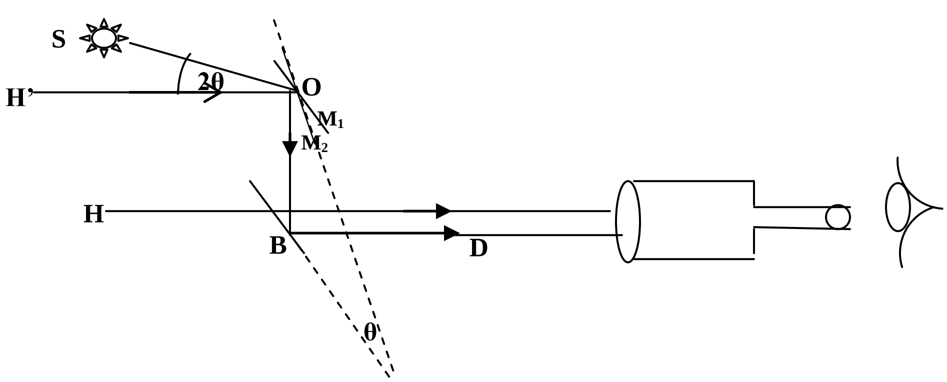
$2\theta = 360 - 2(\alpha + \beta)$

$\therefore 2(\alpha + \beta) = 360^0 - 2\theta$

$\therefore$ Net deviation $= 360^0 - 2\theta$ (*clockwise*) or 2θ anticlockwise.

Principal of the sextant

The sextant is an instrument used in navigation for measuring the angle of elevation of the sun or stars. It consists essentially of a fixed glass B, silvered on a vertical half, and a silvered mirror O which can be rotated about a horizontal axis a small fixed telescope T is connected towards B.



Suppose that the angle elevation of the sun, S is required. Looking through T, the mirror O is turned until the view H of the horizon seen directly through the unsilvered half of B and also the view of it, H seen by successive reflection at O and the silvered half of B are coincident. The mirror O is then parallel to B in the position M_1 and the ray HO is reflected along OB and BD to enter the telescope T. The mirror O is now rotated to a position M_2 until the image of the sun S is seen by successive reflections at O and B is on the horizon H^1 and the angle of rotation Ө of the mirror is noted.

The ray SO from the sun is now reflected in turn from O and B so that it travels along BD, the direction of the horizon and the angle of deviation of the ray is thus angle SOH. But the angle between the mirrors M_2 and B is Ө. Thus from our result for successive reflections at two inclined mirrors, angle SOH = 2Ө. Now the angle of elevation of the sun, S is angle SOH. Hence the angle of elevation is twice the angle of rotation of mirror O and can thus be easily measured from a scale which measures the rotation of O.

Since the angle of deviation after two successive reflections is independent of the angle of incidence on the first mirror, the image of the sun S through T will continue to be seen on the horizon once O is adjusted, no matter how the ship pitches or rolls. This is an advantage of the sextant.

Question:

1. State the properties of images produced by the plane mirrors.

2. With the aid of a ray diagram, show how a plane mirror can form

(i) a real image

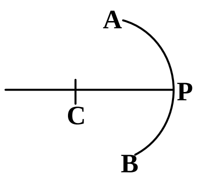
(ii) a virtual image.

REFLECTION AT CURVED MIRRORS

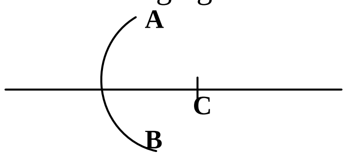
There are two types of curved mirrors namely:

1. concave mirror / converging mirror.
2. convex / diverging mirror.

Concave/converging mirror



Convex/diverging mirror



C is the centre of curvature and is the centre of the sphere of the mirror is part.

P is the pole of the mirror and it is the centre of the mirror surface.

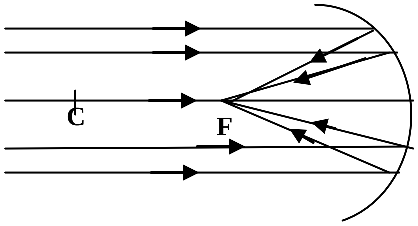
AB is the aperture of the mirror.

The line joining C to P is called the principal axis.

The distance CP is called the radius of curvature.

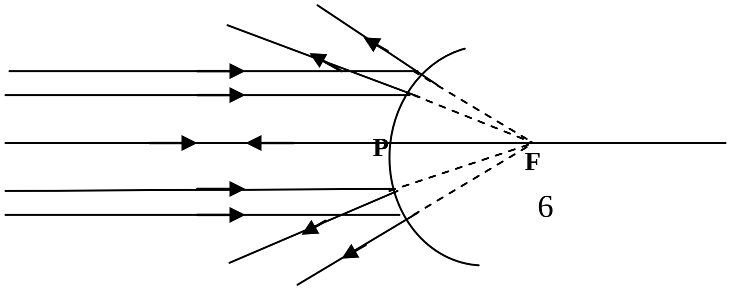
Principal focus

Consider a parallel beam of light incident on a concave mirror and close to the principal axis of the mirror, after reflection the rays converge through F



Point F is called the *principal focus* of the mirror since light actually passes through it.

A narrow beam of rays, parallel and near to the principal axis falling on the convex mirror is reflected to form a divergent beam of appears to come from a point F behind the mirror.



The point F is the principal focus of the convex mirror and it is a virtual focus.

Focal length is the distance between the pole and principal focus.

Hence

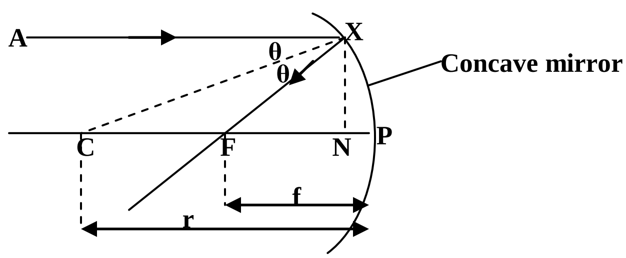
The *principal focus of a concave mirror* is the point through which all rays parallel and close to the principal axis converge after reflection.

The *principal focus of a convex mirror* is the point through which all rays parallel and close to the principal axis appear to diverge from after reflection.

Paraxial rays

Rays which are close to principal axis and make small angles with it are called paraxial rays.

Relationship between the radius of curvature and the focal length



Consider a ray parallel and near the principal axis of a converging mirror, concave mirror after reflection the ray passes through F if a normal line is drawn from X ,it will pass through point C.

$< AXC = < CXF = \theta$ (laws of reflection)

$< AXC =< XCP = \theta$ (alternating angles)

$< XFP = < AXF = 2\theta$ (alternating angles)

For paraxial rays and angles in radians,

$$\tan XCP = \tan \theta = \frac{XN}{CN} \approx \theta (Using \Delta XCN)$$

$$2\theta \approx \tan 2\theta = \frac{XN}{FN}$$

Hence $\frac{XN}{FN} = 2\theta = 2\frac{XN}{CN}$

$$\frac{2}{CN} = \frac{1}{FN}$$

Since X tends to P then N tends to P

$\overline{CN} \simeq \overline{CP}$ and $\overline{FN} \simeq \overline{FP}$

$\frac{2}{CP} = \frac{1}{FP}$

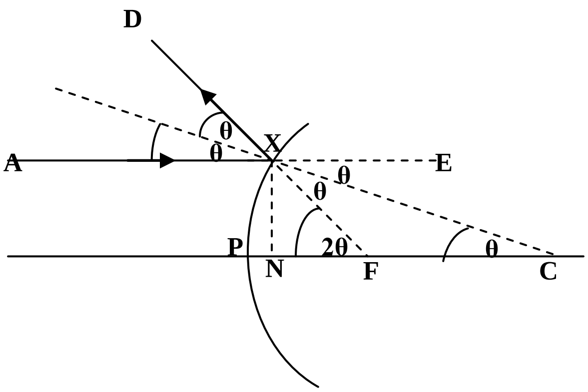
But FP = f

CP = r

$\frac{2}{r} = \frac{1}{f}$

hence *r = 2f*

For a convex mirror



Consider a ray parallel and near the principal axis of a diverging mirror(convex mirror), after reflection the ray appears to be coming from F

If the normal line is drawn through X, it passes through point C.

$< AXB = < BXD$ (laws of reflections)

$< BXD = < FXC$ (vertically opposite angles)

$< NFX = < FXE$ (altemating angles) $= 2\theta$

$$\tan XFN = \tan 2\theta = \frac{XN}{FN}$$

$$\tan XCN = \tan \theta = \frac{XN}{CN}$$

For paraxial rays and angles in radians,

$\theta \approx \tan \theta$

$2\theta \approx \tan 2\theta$

hence $2\theta = \frac{XN}{FN}, \theta = \frac{XN}{CN}$

$$2\theta = \frac{XN}{FN} = 2\frac{XN}{CN}$$

$$\frac{2}{CN} = \frac{1}{FN}$$

$$CN \approx CP$$

$$FN \approx FP$$

Hence

$$\frac{2}{CP} = \frac{1}{FP}$$

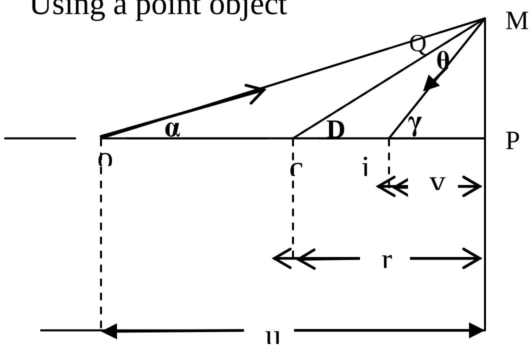
$$CP = r, FP = f$$

Hence $r = 2f$

The Mirror Formula

Concave mirror

Using a point object



Using triangle OCM

$\alpha + \theta = \beta$ (exterior angle properties of triangle)

$\theta = \beta - \alpha$ ....(1)

Using CIM

$\theta + \beta = \gamma$

$\theta = \gamma - \beta$ ....(2)

From equation 1 and 2

$\beta - \alpha = \gamma - \beta$

$\gamma + \alpha = 2\beta$ ....(3)

For paraxial rays and angles in radian

$$\left.\begin{aligned}\alpha &\approx \tan\alpha = \frac{\overline{MP}}{OP} = \frac{\overline{MP}}{U}\left(u \sin g\,\Delta OMP\right)\\ \beta &\approx \tan\beta = \frac{MP}{CP} = \frac{\overline{MP}}{r}\left(u \sin g\,\Delta MCP\right)\\ \gamma &\approx \tan\gamma = \frac{\mathrm{MP}}{\mathrm{IP}} = \frac{MP}{v}\left(u \sin g\,\Delta MIP\right)\end{aligned}\right\} 4$$

Substitute equation 4 in equation 3

$$\gamma + \alpha = 2\beta$$

$$\frac{\overline{MP}}{v} + \frac{\overline{MP}}{u} = 2\left(\frac{MP}{r}\right)$$

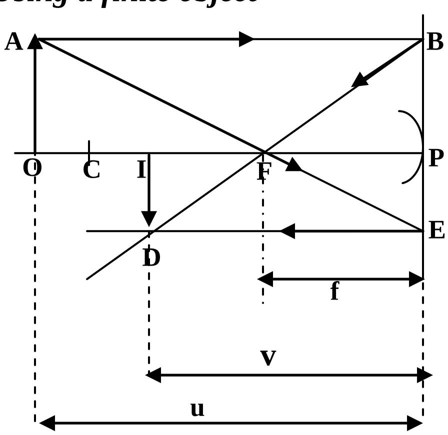
$$\frac{1}{v} + \frac{1}{u} = \frac{2}{r}$$

but $r = 2f$

$$\frac{1}{\mathrm{v}} + \frac{1}{u} = \frac{2}{2f}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Using a finite object



$\Delta\,\mathrm{OAF}$ is similar to $\Delta\,\mathrm{FPE}$

$$\therefore \frac{\overline{\mathrm{PE}}}{\mathrm{OA}} = \frac{\overline{FP}}{OF} = \frac{f}{u - f}$$

$$OF = OP - FP = u - f$$

$$\therefore \frac{\overline{PE}}{OA} = \frac{f}{u - f} \text{.......... ....(1)}$$

ΔIDF is similar to Δ FBP

$$\frac{\overline{\text{DI}}}{\text{BP}} = \frac{FP}{FP} = \frac{v - f}{f}$$

$$\therefore \frac{\overline{DI}}{bp} = \frac{v - f}{f} \text{..........} \quad \text{..........} \quad \text{..(2)}$$

But

$$\frac{\overline{DI}}{\overline{\overline{BP}}} = \frac{\overline{PE}}{\overline{\overline{OA}}}$$

$$= \frac{\overline{PE}}{OA} = \frac{v - f}{f}$$

From equation (1) and (2)

$$\frac{f}{u - f} = \frac{v - f}{f}$$

$$\frac{f}{u - f} = \frac{v - f}{f}$$

$$f^2 = (v - f)(u - f)$$

$$f^2 = uv - vf - uf + f^2$$

$$f^2 - f^2 = uv - vf - uf$$

$$uf + vf = uv$$

$$(\text{divide thru by uvf})$$

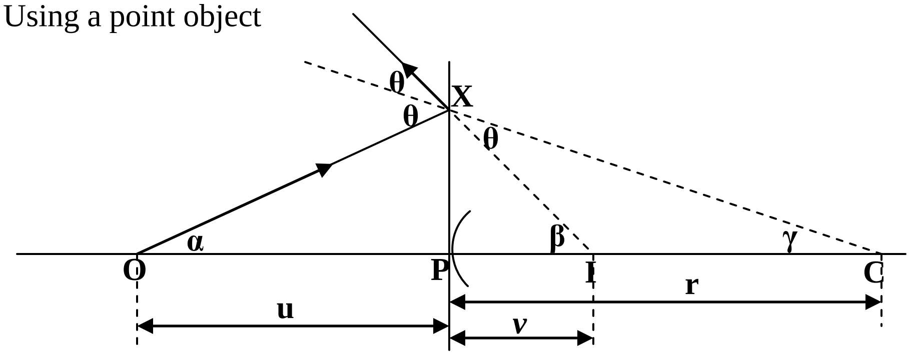
$$= \frac{uf + vf}{uvf}$$

$$\frac{1}{f} = \frac{uf}{uvf} + \frac{uf}{uvf}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

CONVEX MIRROR

Using a point object



Using triangle OXI

$\alpha + \beta = 2\theta$.......... .(1)

Using ΔIXC

$\theta + \gamma = \beta$

$\theta = \beta - \gamma$.......... ....(2)

Substitute 2 in equation 1

$\alpha + \beta = 2(\beta - \gamma)$

$\alpha + 2\gamma = 2\beta - \beta$

$\alpha + 2\gamma = \beta$.........(3)

For paraxial rays and angles in radian,

$$\alpha \approx \tan\alpha = \frac{XP}{OP} = \frac{XP}{u} \left(U \ \sin \ g \Delta XOP \right)$$

$$\beta \approx \tan\beta = \frac{XP}{IP} = \frac{XP}{v} \left(\text{Using} \quad \Delta \text{XPI} \right)$$

$$\gamma \approx \tan\gamma = \frac{XP}{CP} = \frac{XP}{r} \left(\text{Using} \quad \Delta \text{XPC} \right)$$

$\alpha + 2\gamma = \beta$

$$\frac{XP}{u} + 2\left(\frac{XP}{r}\right) = \frac{XP}{v}$$

$$\frac{1}{u} + \frac{2}{r} = \frac{1}{v} \text{.......... ....(5)}$$

Introducing a sign convention so that the distances are given positive or negative sign, the same equation is obtained for both concave and convex mirrors, irrespective of whether the objects and images are real or virtual.

We shall adopt the "*REAL IS POSITIVE*" rule which states that the real object or image distance is positive and a virtual object or image distance is negative.

The focal length of a concave mirror is positive and that of a convex mirror is negative.

The radius of curvature takes the same sign as focal length

For convex mirror

r is negative

v is positive

$$\frac{1}{u} - \frac{2}{r} = \frac{-1}{u} \text{ from equetion (5)}$$

$$\frac{1}{\mathrm{u}} + \frac{1}{v} = \frac{2}{v} \text{ but } \mathrm{r} = 2\mathrm{f}$$

$$\frac{1}{\mathrm{u}} + \frac{1}{v} = \frac{2}{2f}$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Question

Derive expression for the mirror formula using a finite object in front of a convex mirror

Uses of curved mirrors

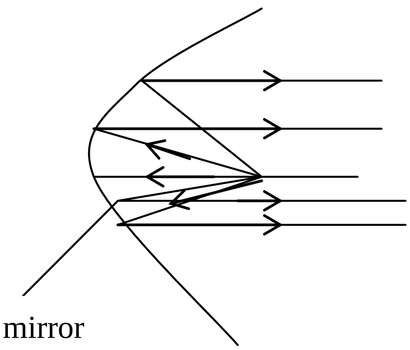
The convex mirror is used as a driving mirror

- Provides an upright image
- Has a wide field of view

Concave mirror is used as a shaving mirror

Solar reflection or telescope

PARABOLIC MIRROR USED AS A SEARCH MIRROR



If a small lamp is placed at the focus F, of the concave mirror, it follows from the principle of reversibility of light that rays striking the mirror round a small area about the pole are reflected parallel.

But these rays from the lamp which strike the mirror at points well away from pole will be reflected in different directions because a wide parallel beam is not brought to a focus at F

The beam of light reflected from the mirror thus diminishes in intensity as its distance from the mirror increases and a concave spherical mirror is hence useless as a search mirror.

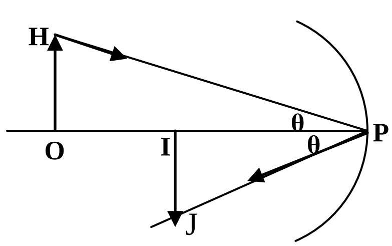
A mirror whose section is the shape of a parabola is used in search lights. Parabolic mirror has the property of reflecting with wide beam of light from a lamp at its focus as a perfectly parallel beam in which case the intensity of the reflected beam is practically undiminished as the distance from the mirror increases.

Linear /lateral or transverse magnification

It is the ratio of the height of image to the height of the object.

$$m = \frac{\text{height of image}}{\text{height of object}}$$

Consider the formation of an image of finite object by spherical mirrors



ΔOHP is similar to Δ IJP

$$\frac{IJ}{OH} = \frac{IP}{OP} \text{ but } \frac{\text{IJ}}{\text{OH}} = m$$

$$m = \frac{IJ}{OH} = \frac{IP}{OP} = \frac{\text{Image distance from mirror}}{\text{object distance from mirror}}$$

From the mirror formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

multiply t hru by v

$$\frac{\text{v}}{\text{f}} = \frac{v}{u} + 1$$

$$\text{but } \frac{\text{v}}{\text{u}} = m$$

$$\frac{v}{f} = m + 1$$

$$\therefore \quad \boxed{m = \left(\frac{v}{f} - 1\right)}$$

Example

1. A converging mirror produces an image whose length is 2.5 times that of the object. If the mirror is moved through a distance of 5cm towards the object, the image formed is 5 times as long as the object. Calculate the focal length of the mirror.

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{u}{f} = \frac{u}{v} + \frac{u}{u}$$

$$\frac{u}{f} = \frac{1}{m} + 1$$

$$m_1 = 2.5 \quad \text{u} = \text{x}$$

$$\frac{\text{x}}{\text{f}} = \frac{1}{2.5} + 1$$

when $m_2 = 5 \quad u = x - 5$

$$\frac{x - 5}{f} = \frac{1}{5} + 1$$

$$\frac{x}{f} - \frac{5}{f} = \frac{6}{5}$$

$$\frac{1}{2.5} + 1 - \frac{5}{f} = \frac{6}{5}$$

$$\frac{5}{f} = \frac{1}{2.5} + 1 - \frac{6}{5}$$

$$f = 25cm$$

2. A concave mirror forms a real image which is 3 times the linear size of the real object. When the object is displaced through a distance d, the real image formed is now 4 times the linear size of the object if the distance between two image position is 20cm; find

i. The focal length of the mirror
ii. The distance d

$m_1 = 3$

$m_2 = 4$

$$m = \frac{v}{f} - 1$$

$$\frac{v}{f} = m + 1 \qquad \frac{v_2}{f} = m + 1$$

$$\frac{v_1}{f} = 3 + 1 \qquad \frac{v_2}{f} = 4 + 1$$

$$\frac{v_1}{f} = 4 ....(i) \qquad \frac{v_2}{f} = 5.......... ....(i)$$

(ii) __(i)

$$\frac{v_2}{f} - \frac{v_1}{f} = 5 - 4 \qquad \frac{v_2 - v_2}{f} = 1$$

$$20 = f$$

$$f = 20\ cm$$

(ii)

$v_1 = 4 \times 20$ $\quad$ $v_2 = 5 \times 20$

$= 80\ cm$ $\quad$ $100cm$

$\therefore$

$\frac{1}{u} = \frac{1}{20} - \frac{1}{80}$ $\quad$ $\frac{1}{u} = \frac{1}{20} - \frac{1}{100}$

$\frac{1}{u} = \frac{3}{80}$ $\quad$ $\frac{1}{u} = \frac{1}{25}$

$u = 26.67$ $\quad$ $u = 25\ cm$

$$d = 26.67 - 25$$
$$= 1.67\ cm$$

Exercise

1. A real image is formed 40 cm from a spherical mirror, the image being twice the size of the object. What kind of mirror is it and what is the radius of the curvature.
2. An object is 4cm high. It s desired to form a real image 2cm high and 96cm from the object. Determine the type of mirror required and focal length of the mirror.
3. A dentist holds a concave mirror of focal length 4cm at a distance of 1.5 cm from the tooth. Find the position and magnification of the image which will be formed.

Determining the focal length of curved mirrors

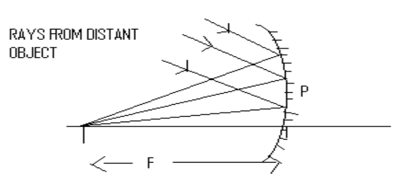
Concave mirror.

i)*focusing distant object*

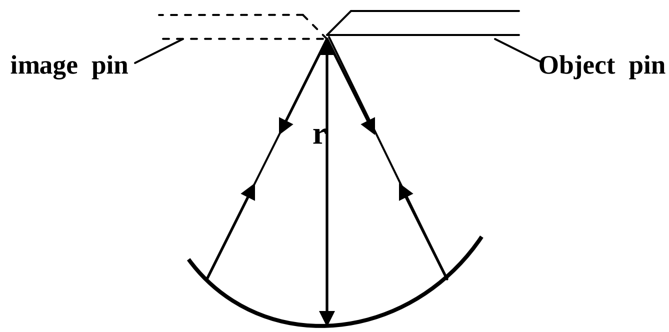
Light from a distant object such as a tree is focused on the screen.

Distance between the image and the pole of the mirror is measured.

It is equal to the focal length.



ii) *By determining first the radius of curvature.*



A concave mirror is placed horizontally on a bench. An optical pin is clamped horizontally on a tripod stand so that the tip lies along the principal axis of the mirror.

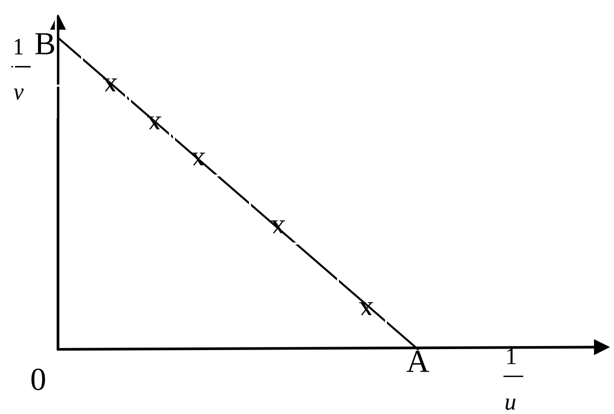
The position of the pin is adjusted until the position is obtained where it coincides with its image and there is no parallax between the two, i.e. there is no relative motion between the object and the image when the observer moves the head from side to side or up and down.

The distance r of the pin from the pole is measured and focal length determined, $\left(f = \frac{r}{2} \right)$

iii) *Using the mirror formula.*

Several values of image distance v corresponding to different values of the object distance u are found using other an illuminated object and screen ,no parallax method.

A graph of 1/v and 1/u is plotted

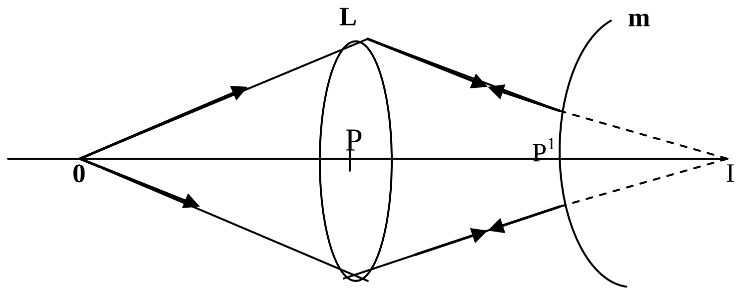


The intercepts OA and OB are determined and the focal length is calculated from

$$f = \frac{1}{2}\left(\frac{1}{OA} + \frac{1}{OB} \right)$$

Determine the focal length of the convex mirror

Using a convex lens



The real image I of an object O as formed by a convex lens is located, the distance PI is measured. The convex mirror is placed between P and I. the portion of the mirror is adjusted until the image of O coincides with O itself, this happens when light rays from O are incident normally in the convex mirror after refraction by the lens, the distance PP^1 is measured, the radius of curvature is

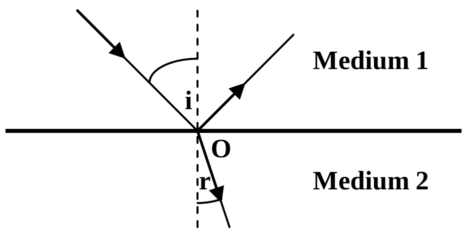
$r = PI - PP^1$ Hence $f = \frac{r}{2}$

Refraction is the change in speed of propagation of light due to change in optical density.

When light propagating in free space is incident in medium, the electrons and protons interact with the electric and magnetic fields of the light wave. This result in the slow down of a light wave.

Law of refraction

When light passes from one medium to another, say from air glass part of it is reflected back into the previous medium and the rest passes through the second medium with its direction of travel changed.



Law 1. The incident ray, refracted ray and the normal at point of incidence all lie on the Same plane

Law 2. For any two particular media, the ratio of the sine of angle of incidence to sine of angle of refraction is constant.

i.e. $\frac{\sin i}{\sin r} = constant$

The constant ratio $\frac{\sin i}{\sin r}$ is called the refractive index for light passing from the first to second medium

$$hence \quad {}_1n_2 = \frac{\sin i}{\sin r}$$

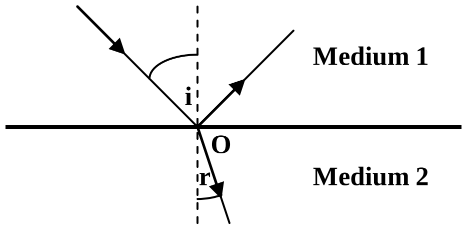
If medium 1 is a vacuum/air, we refer the ratio as the absolute refractive index of medium 2, denoted by n_2

$${}_1n_2 = \frac{V_1}{V_2} = \frac{\text{speed of light in medium 1}}{\text{speed of light in medium 2}}$$

$$n_2 = \frac{c}{V_2}$$

$$(\text{speed of light in vacuum, } c = 3 \times 10^8)$$

Refractive index relationships



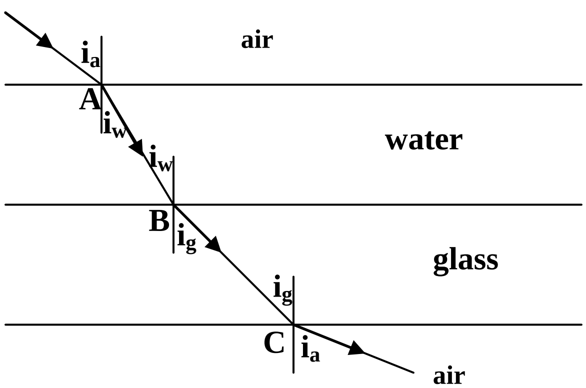
Consider ray of light incident at a point O on the plane boundary between medium 1 and 2

$${}_1n_2 = \frac{\sin i}{\sin r}$$

The principal of reversibility of light indicates that when light passes from B to O, it will be refracted along OA.

$${}_2n_1 = \frac{\sin r}{\sin i}$$

$${}_1n_2 \times {}_2n_1 = 1 \therefore \boxed{{}_1n_2 = \frac{1}{{}_2n_1}}$$



At A, $${}_an_w = \frac{\sin i_a}{\sin i_w} \text{.......... ....} (i)$$

At B, $${}_w\cap_g = \frac{\sin iw}{\sin ig} \text{.......... ....(} ii)$$

At C, $_{g}n_{a} = \frac{\sin i_g}{\sin i_a}$.......... ....(iii)

Eqn (i) x eqn (ii) X eqn (iii), you get

$_{a}n_{w} \times {}_{w}n_{g} \times {}_{g}n_{a} = 1$

From equation (i) $\sin i_a = {}_{a}n_{w} \sin i_w$.......... ......$(iv)$

Equation (iii) $\sin i_a = \frac{\sin i_g}{_{g}n_{a}}$

But $_{a}n_{g} = \frac{1}{_{g}n_{a}}$

$\sin i_a = {}_{a}n_{g} \sin i_g$.......... ....$(v)$

From equation (iv) and (v)

$\sin i_a = {}_{a}n_{w} \sin i_w = {}_{a}n_{g} \sin i_g$

Considering absolute refractive indices of air, water and glass

$n_a \sin i_a = n_w \sin i_w = n_g \sin i_g$

In general

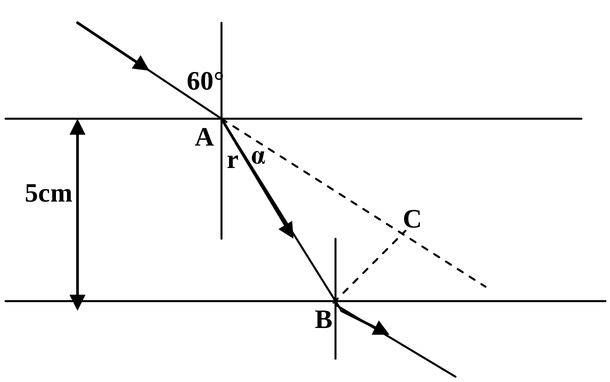
$\cap \sin i = \text{constant}$

Where i is the angle in the medium with absolute refractive index n

Examples

1. A ray of light is incident at angle of 60° on one surface of the glass plate 5cm thick and of refractive index 1.5

   The medium on either side of the plate is 1.find the transverse displacement between the incident and emergent rays.



$$n_g = \frac{\sin i}{\sin r}$$

$$1.5 = \frac{\sin 60}{\sin r}$$

$$r = 35.26^0$$

$$\alpha = 60 - r = 60 - 35.26 = 24.74^o$$

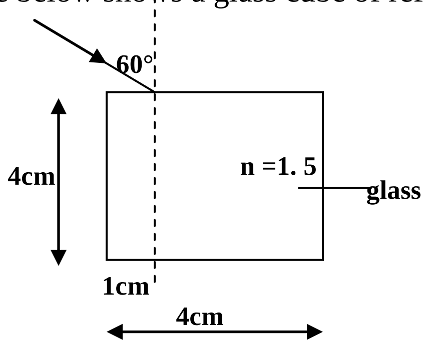
$$AB = \frac{5}{\cos r} = 6.1cm$$

$$BC = AB \sin \alpha = 6.1 \times \sin 24.74 = 2.55\,cm$$

Assignment

1. Monochromatic light is incident on a block of transparent material placed in a vacuum. The light is refracted through an angle θ if the block has refractive index $\cap$ and is of thickness t, show that the light takes a time $\frac{nt \sec \theta}{c}$ to emerge from the block where c is the speed of light in a vacuum.

2. A beam of light is incident on a surface of water at an angle of 30° with the normal to the surface. The angle of refraction in water is 22°.Find the speed of light in water if it is 3 x 10^8 ms^{-1}.
3. The figure below shows a glass cube of refractive index 1.5. Find the path of light

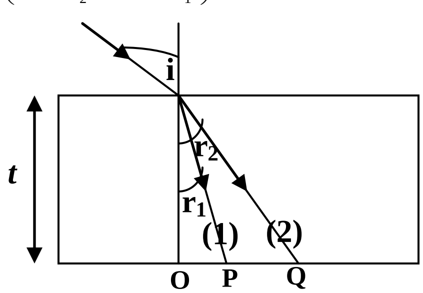


4. Light consisting of two colors is incident from air on a glass block

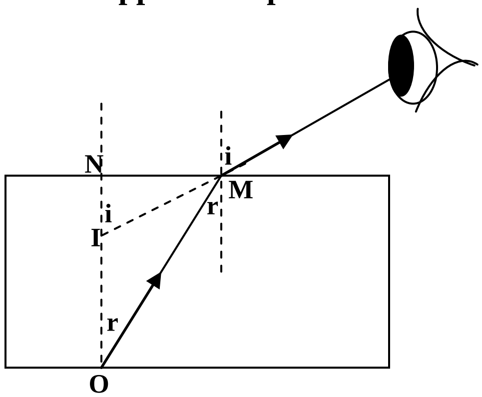
The speeds of ray 1 and ray 2 glasses are v_1 and v_2. Respectively.

Show that the distance PQ given by

$$PQ = tc^{-1} \sin i \left(\frac{v_2}{\cos r_2} - \frac{v_1}{\cos r_1} \right)$$ Where c is speed of light in air.



Real and Apparent depth



Rays from O are bent a way from the normal at the glass air boundary could appear to come from I, the image of O.

$$n_g = \frac{\sin i}{\sin r}$$

Using Δ OMN , $\sin r = \frac{\overline{\mathrm{NM}}}{\mathrm{OM}}$

Using Δ NIM, $\sin i = \frac{\overline{NM}}{IM}$

$$n_g = \frac{\sin i}{\sin r} = \frac{\overline{NM}}{IM} \div \frac{\overline{NM}}{OM} = \frac{\overline{OM}}{IM}$$

Viewing the object O directly above it

$\overline{MI} \approx NI$

$\overline{OM} \approx \overline{ON}$

$$\therefore n_g = \frac{\overline{ON}}{\overline{NI}} = \frac{\text{Re}\ aldepth}{apparentde\quad pth} = \frac{t}{apparentde\quad pth}$$

Apparent depth = $\frac{t}{n}$

displacement, d = real depth - apparent depth

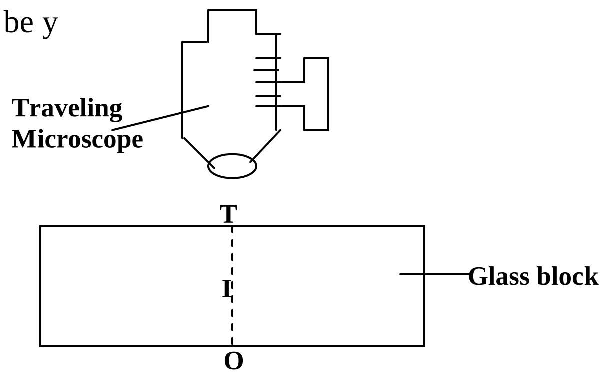
$$d = t - \frac{t}{n}$$

$$= t\left(1 - \frac{1}{n}\right)$$

Measurement of refractive index using real and apparent depth method (both solids and liquids)

A traveling microscope is focused on a pencil dot O on a sheet of white paper lying on a bench, and the reading, x, on the microscope scale noted.

If the refractive index of glass is required, a block of the material is placed over the dot, and the microscope refocused on the image I of O as seen through the block, Let the reading of the microscope be y



The microscope is focused on the top T of the visible by lycopodium powder. suppose the reading now is z

$$n = \frac{\text{real depth}}{\text{apparent depth}} = \frac{|x - z|}{|y - z|}$$

For a liquid in a beaker, similar procedures are followed.

1.A microscope is focused on a scratch on the bottom of the beaker. Turpentine is poured into the beaker to depth of 4cm and it is found to raise the microscope through a vertical distance of 1.28cm to bring the scratch back into focus. Find the refractive index of turpentine.

$$n = \frac{\text{real depth}}{\text{apparent depth}} = \frac{4}{4 - 1.28} = \frac{4}{2.72} = 1.47$$

2. A microscope is first focused on a scratch on the inside of the bottom of an empty glass dish. water is then poured in and it is found that the microscope has to be raised by 1.2cm for refocusing. Chalk dust is sprinkled on the surface of water and this dust comes into focus when the microscope is raised an additional 3.5cm.

Find the refractive index of water

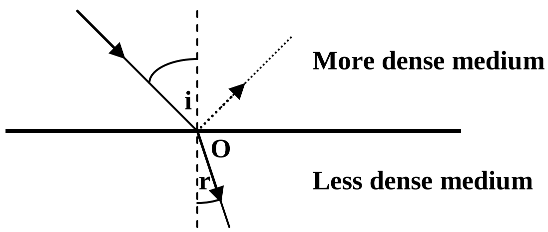
Displacement = 1.2,   apparent depth =3.5,   real depth = 1.2+ 3.5= 4.7 cm

$$n = \frac{\textit{real depth}}{\textit{apparent depth}} = \frac{4.7}{3.5}$$

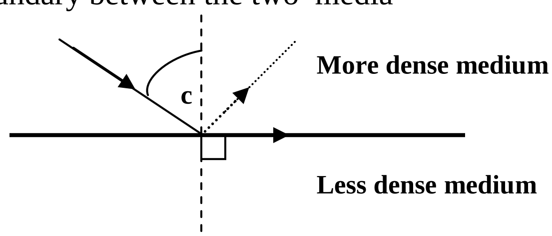
$$n = 1.34$$

TOTAL INTERNAL REFLECTION AND CRITICAL ANGLE

Consider monochromatic light propagating from a dense medium and incident on a plane boundary with less dense medium at a small angle of incidence. Light is partly reflected and partly refracted.



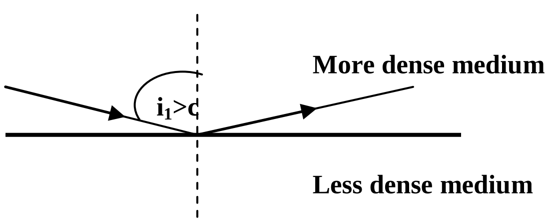
As the angle of incidence is increased gradually, a stage is reached when the refracted ray grazes the boundary between the two media



The angle of incidence **c** is called the critical angle.

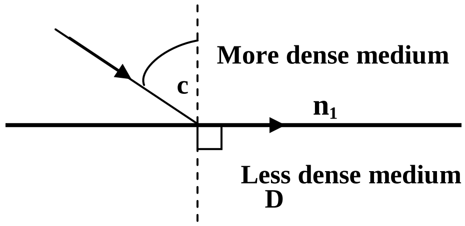
Hence *critical angle* is the angle of incidence in a more dense medium which makes the angle of refraction in a less dense medium 90°.

When the angle of incidence is increased beyond the critical angle, the light is totally internally reflected in the dense medium.



Total internal reflection is said to have occurred.

Relationship between critical angle, c, and refractive index, n.



Using Snell's law

$$n_1 \sin c = n_2 \sin 90$$

$$\sin c = \frac{n_2}{n_1}$$

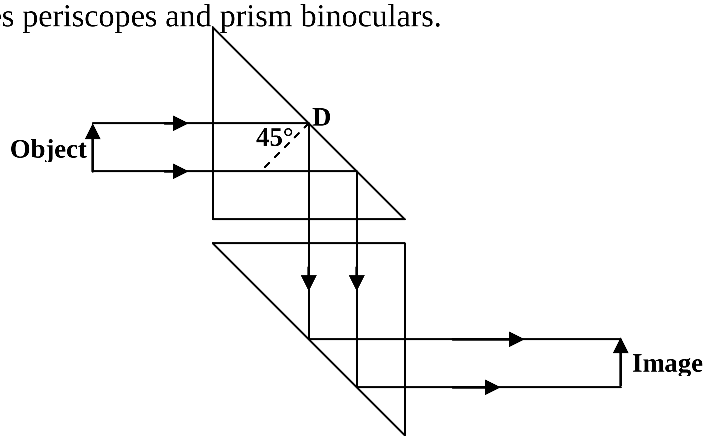
If the lens dense medium is air / vacuum,

$$\sin c = \frac{1}{n}$$

Applications of total internal reflection

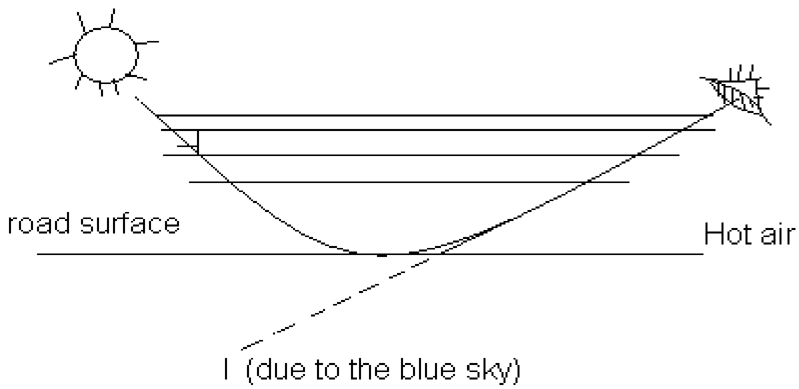
Prism periscopes

The angle of incidence at D is equal to 45 which is greater than the critical angle hence the light is totally internally reflected at D and emerges at right angles to BC. Two such prisms are used in submarines periscopes and prism binoculars.



Mirage

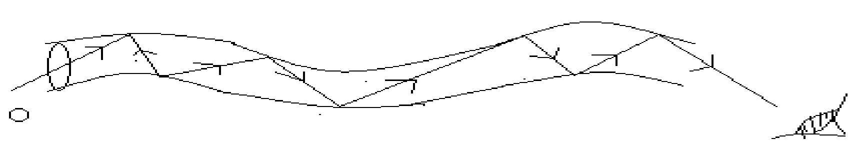
On a hot day (clear), the air near a road gets heated and the density of air decreases as the road is approached. A ray of light from the blue sky is progressively refracted a way from the normal until the critical angle is reached when light is totally internally reflected.



An observer at E sees an inverted image of object O as through there was a pool of water on the road due to the blue colour of the sky.

3. **Optical fibres (light pipes)**

Light can be trapped by total internal reflecting using a bent glass rod. The beam is reflected from side practically count loss and emerges only at the end of the rod where it strikes the surface almost normally.



A single, very thin solid glass fibre behaves in the same way and if several thousands are topped together a flexible light pipe is obtained. Optical fibres are also applied in communication systems where they are used to transmit information using a modulated laser beam.

Disadvantage

There is leakage of light at places of contact between the glass (because light is absorbed by glass).it can be avoided by coating each fibre with glass of lower refractive index,than its own hence encouraging total internal reflection.

Advantages of the prism periscope over the mirror periscopes

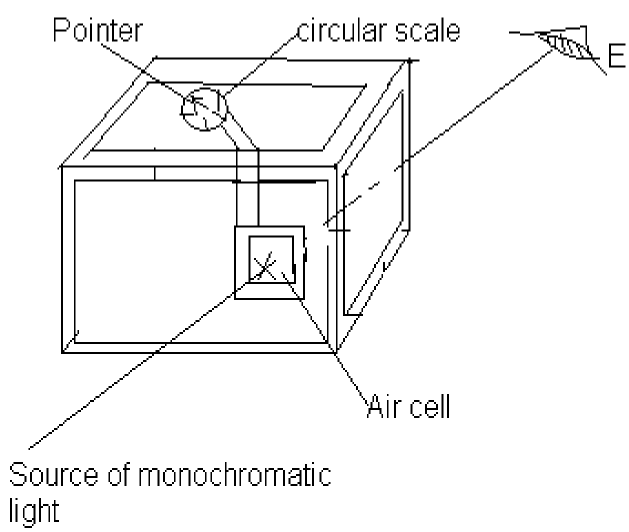
1. Mirrors form multiple images due to partial reflections in the body of the mirror while in prisms there is a single reflection on the surface i.e. the prisms form sharp images.
2. Some light is absorbed at the reflecting surface of the mirror therefore images are less spread but there is total reflection at surface of the prism therefore the images are brighter.
3. With time, mirrors tarnish but prism surface do not tarnish therefore a long lasting.

Disadvantages

The prisms are more expensive and bulkier than mirrors.

A periscope is used to observe objects over obstacles.

Determining refractive index of a liquid using the air cell method

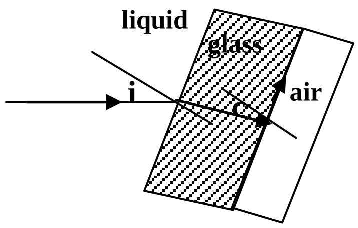


Two thin plane parallel glass plates such as microscope slides are cemented together containing a thin film of constant thickness, hence forming an air cell.

The liquid whose refractive index is required is poured inside a glass vessel. the air cell x is placed in the liquid. Monochromatic light is directed normal to x and is observed from the opposite side E.

The air cell is rotated until light is suddenly cut off from E revolution of the pointer on the circular scale is noted.

The air cell is rotated in the opposite direction until light is again suddenly cut off from E.the reading on the circular scale is noted. the angle Ɵ between the two positions is obtained. the refractive index of the liquid is calculated from $n_l = \frac{1}{\sin\left(\theta/2\right)}$



$$n_l \sin i = n_g \sin c = n_a \sin 90$$

$$n_l \sin i = n_a \sin 90 = 1$$

$$n_l = \frac{1}{\sin i}$$

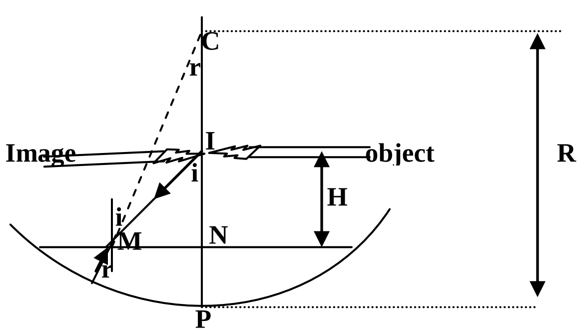
$$i = \theta/2$$

Determining refractive index of a liquid using a concave mirror

A concave mirror is placed facing up on a bench. An optical pin is clamped horizontally in a retort stand so that its apex is a long the principal axis of the mirror. The height of the pin above the mirror is adjusted until the pin coincides with its image and there is no parallax. The height R of the pin above the pole of the mirror is noted.

A little quantity of the liquid whose refractive index is required is dropped in the mirror. The height of the pin is adjusted until the pin again coincides with its image.

The height ***H*** of the pin above the liquid surface is measured.



At M, $n = \frac{\sin i}{\sin r}$ ………………………………………..(1)

Using Δ MIN, $\sin i = \frac{NM}{IM}$ ……………………………….(2)

Using Δ MCN, $\sin r = \frac{NM}{CM}$ ……………………………….(3)

Substitute (2) and (3) in equation (1)

$$n = \frac{NM}{IM} \div \frac{NM}{CM} = \frac{CM}{IM}$$

Observing directly from P

IM $\approx$ IN

CM $\approx$ CN

$$n = \frac{CN}{IN} = \frac{R - d}{H}$$

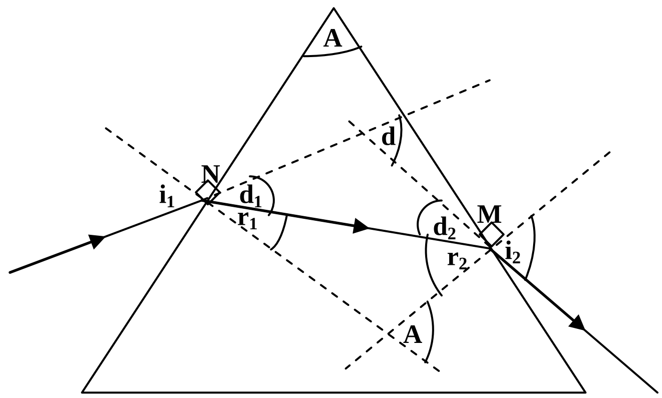
Where d = depth of the liquid.

Example

1. A liquid placed on a concave mirror to a depth of 2cm.An object held above the liquid coincides with the image when it is 5cm from the pole of the mirror. if the radius of curvature of mirror is 6cm,calculated the refractive index

$$n = \frac{R - d}{H} = \frac{6 - 2}{5 - 2} = \frac{4}{3} = 1.33$$

Refraction by glass prism



Angle A = refracting angle of the prism

Deviation at N, $d_1 = i_1 - r_1$

Deviation at M, $d_2 = i_2 - r_2$

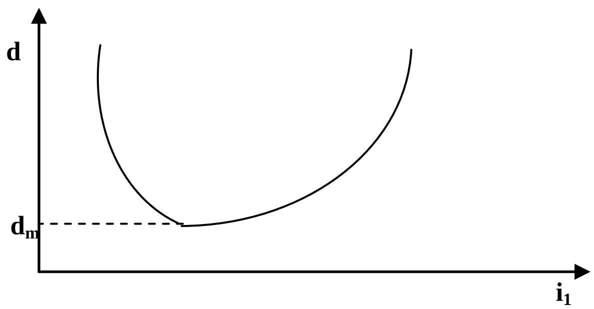
Net deviation $d = d_1 + d_2$

$$d = i_1 - r_1 + i_2 - r_2$$

but $A = r_1 + r_2$

Hence $d = i_1 + i_2 - A$ …………….(1)

If d is measured and angle of incidence i_1 is varied, and a graph of d against i_1 plotted you get,



d_m is Minimum angle of deviation.

Manimum deviation occur when light passes through the prism symmetrically

i.e. when $i_1 = i_2 = i$ and $r_1 = r_2 = r$

When d = d_m

From (i) $d_m = 2i - A$

$$i = \frac{A + d_m}{2}$$

also $r = \frac{A}{2}$

at N, $n_a \sin i = n_g \sin r$

Hence

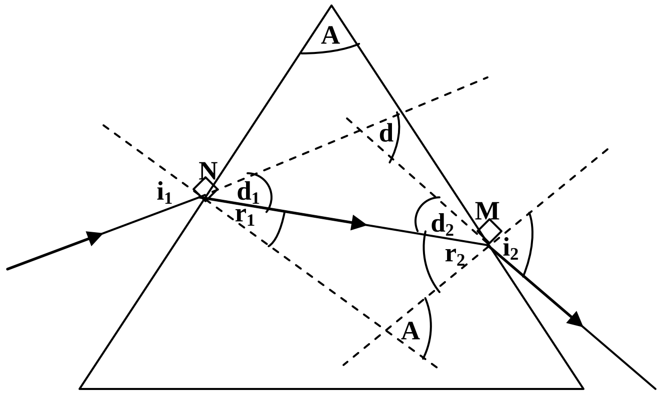
$$\sin\left(\frac{A + d_m}{2}\right) = n_g \sin\left(\frac{A}{2}\right)$$

If the prism is surrounded by a medium of refractive index n_1

$$n_1 \sin\left(\frac{A + d_m}{2}\right) = n_g \sin\left(\frac{A}{2}\right)$$

Deviation of light by a small angle prism

Consider monochromatic light incident almost normally on a small angle prism in air. The angle of the prism is assumed to be so small that sin A $\approx$ A with A in radians



Deviation, $d = i_1 + i_2 - A$ ……………………(i)

At N, $n = \frac{\sin i_1}{\sin r_1}$ --------------------------------- (2)

At M, $n = \frac{\sin i_2}{\sin r_2}$ --------------------------------- (3)

For small angle of i_1 , i_2 , r_1 , r_2 in radians

$\sin i_1 \approx i_1$, $\sin r_1 \approx r_1$

$\sin i_2 \approx i_2$, $\sin r_2 \approx r_2$

From (2) $n = \frac{i_1}{r_1}, \Rightarrow i_1 = nr_1$ ----------------------------- (4)

From (3) $n = \frac{i_2}{r_2}, \Rightarrow i_2 = nr_2$ ------------------------------- (5)

Substitute equations (4) and (5) in (1) and simplify

$d = i_1 + i_2 - A$

$d = nr_1 + nr_2 - A$

but $A = r_1 + r_2$

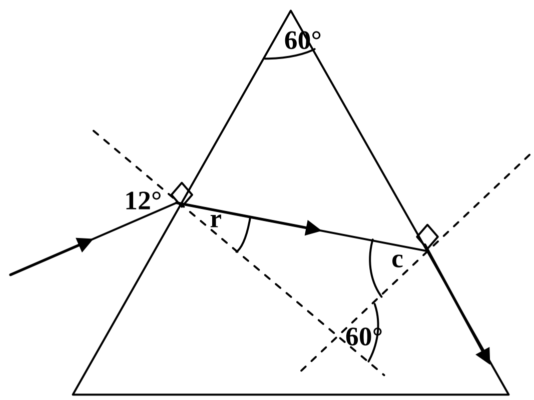
$$d = n(r_1 + r_2) - A = A(n - 1)$$

$$d = A(n - 1)$$

Examples

1. Light propagation in air is incident at 12° on a glass prism of refractive index 1.54 and refracting angle 60 ° as show. The emergent beam glass faces of the prism in contact with the liquid. Find (i) refractive index of liquid

(ii) deviation produced by the prism.



Solution

i) $n_a \sin i = n_g \sin r$

$$n_a \sin 12 = 1.54 \sin r$$

$$\sin 12 = 1.54 \sin r$$

$$r = 7.76^0$$

$$r + c = 60^0$$

$$c = 60 - 7.76 = 52.24^0$$

$$n_g \sin c = n_l \sin 90$$

Hence $1.52 \sin c = n_l$

$$n_l = 1.20$$

ii) $d = i_1 + i_2 - A$

$$d = 12 + 90 - 60 = 42^0$$

2. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive indices 1.54 and 1.48 for the two wave lengths. Find the angular separation of the two wave's length after refraction by the prism.

$d_1 = A(n_1 - 1)$

$d_1 = 5(1.54 - 1) = 2.7^0$

$d_2 = A(n_2 - 1)$

$d_2 = 5(1.48 - 1) = 2.4^0$

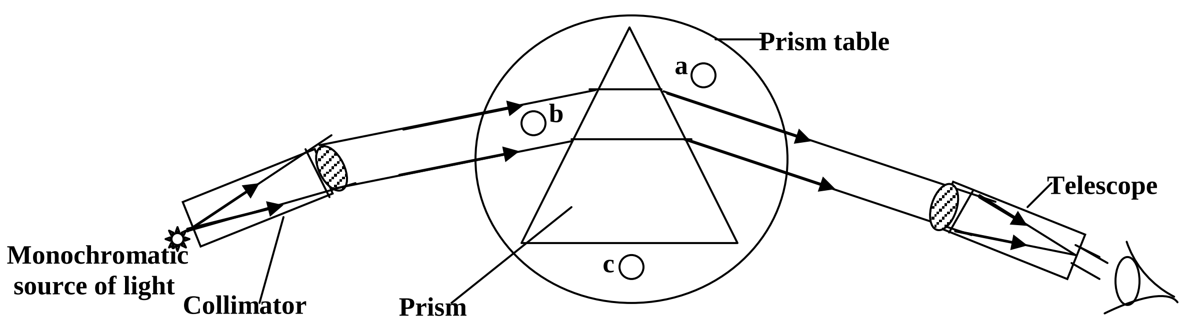
Angular separation, $d = d_1 - d_2 = 2.7 - 2.4 = 0.3°$

Exercise

1. A ray of light just undergoes total internal reflection at the second plane of the prism of refracting angle 60° and refractive index 1.5 what is its angle of incidence on the face

2. A ray of monochromatic light is incident at an angle of 30° on a prism of which the refractive index 1.52.What is the maximum refracting angle of the prism it light is just to emerge from the opposite face.(60.34°)
3. Calculate the critical angle for a glass air surface if a ray of light which is incident in air is deviated through 15.5° when its angle of incidence is 40°.
4. Calculate the angular separation of the red and violet rays which image from a 60° glass prism when a ray of white light is incident on the prism at an angle of 45°. Glass has a refractive index of 1.64 for red light and 1.66 for violet light.

Measurement of refractive index of glass in form of a prism using prism spectrometer



a, b, c are leveling screw

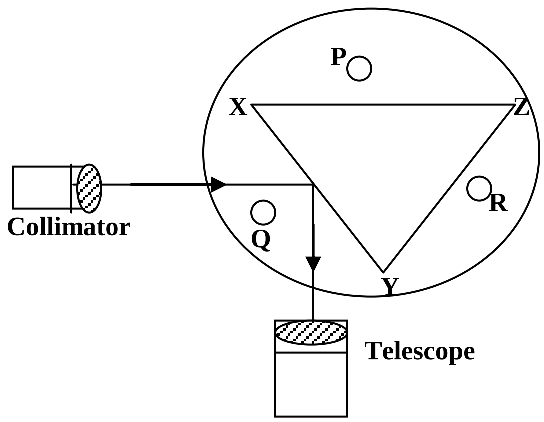
Initial adjustments:

i) The eye piece of the telescope is adjusted so that the cross wires are in sharp focus. The telescope is pointed at distant objects and the telescope objective is adjusted so that the image of the image of the distant object is clearly in focus at the cross wires. Through this adjustment the telescope is set to receive parallel light.

ii) The prism is removed and the collimator is moved in or out of the collimator tube until its image as seen through the telescope is in sharp focus at the cross wires. through this adjustment the collimator is set to produce parallel light.

iii) Leveling of the prism table

The prism is placed on the table in such a way that one refracting face XY is perpendicular to the line joining the leveling screws Q and R

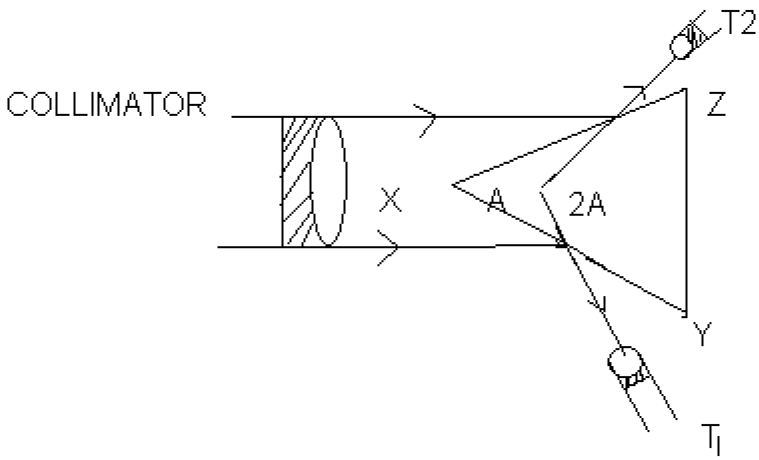


The telescope is turned through 90° and the table turned so that light reflected by XY enters the telescope. the image of the slit of the collimator formed at the cross wires is observed. If the images are not at the centre of the cross wires leveling screw Q is Adjusted until the image is in the centre of the field of view.

The table is then turned so that the refracting surface XZ reflects light into the telescope. The leveling screw P is adjusted if necessary so that the image of the Slit is in the centre of the field of view. The prism table is now level and the spectrometer is ready for use.

Measurement of the refracting angle A

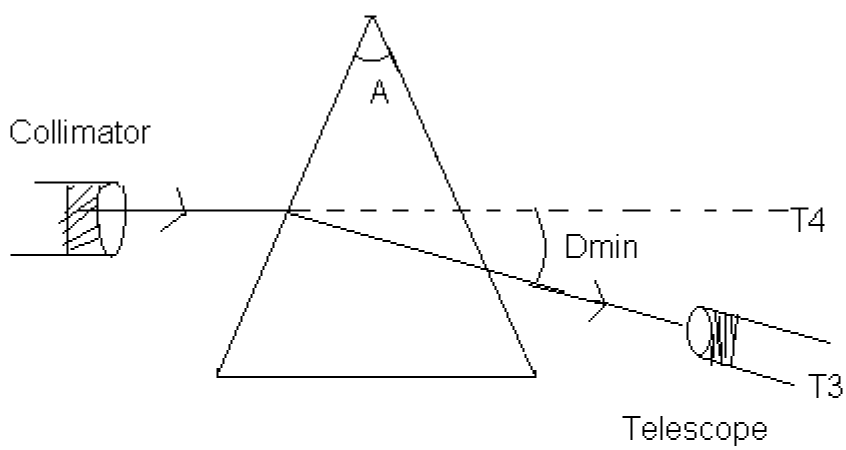
The prism table is turned so that the refracting angle of the prism faces the collimator. The telescope is turned to position T_1 to receive light reflected by the refracting surface XY.An image of the slit is observed at the centre of the cross wires. The reading θ_1 on the circular scales of the spectrometer are noted. The telescope is then moved to position T_2 to receive light reflected from the refracting surface XZ of the prism. The new reading θ_2 on the circular scales of the spectrometer are noted.



The value of $\theta = (\theta_2-\theta_1)$ is obtained and is equal to twice the refracting angle, A or $\theta = 2A$

Measurement of minimum deviation D

The prism is placed with the refracting angle pointing a way from the collimator as shown below. The prism table and telescope are moved in such a direction that the image of the slit remains centered on the cross wires. A stage is reached where the image begins to move in the opposite direction to that in which it had been moving. This is the position of minimum deviation, call it T_3 the readings D_1 on the scales of the spectrometer are taken.



The prism is then removed and the telescope turned to position T4 to receive the undeviated beam. the readings D_2 on the scales of the spectrometer are taken.

The value of $D = D_2 - D1$ is calculated and is the angle of minimum deviation.

The refractive index of the material of the prism is calculated from

$$n = \frac{\sin\left(\frac{D + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Lenses

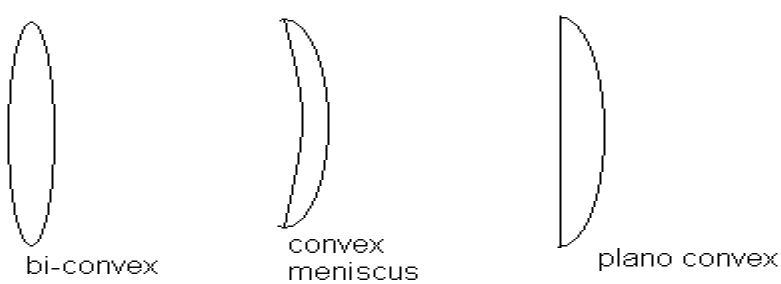
Types of lenses:

There are two types of lenses.

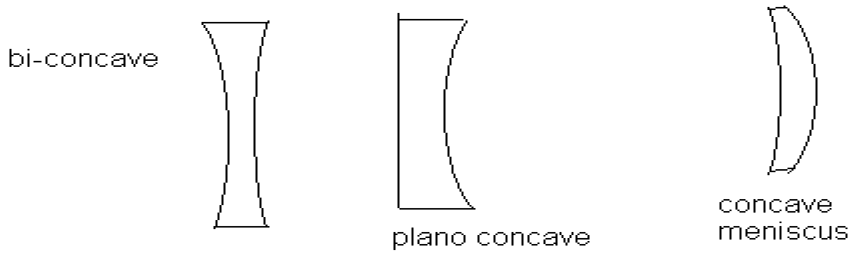
i) convex / converging

ii) Concave / diverging.

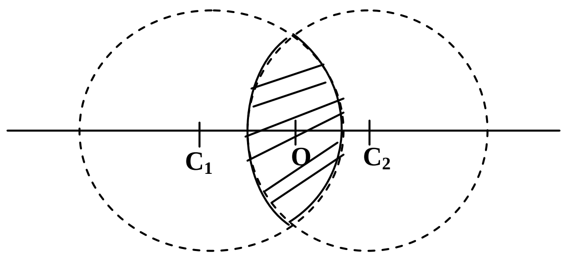
Convex lens



Concave lens



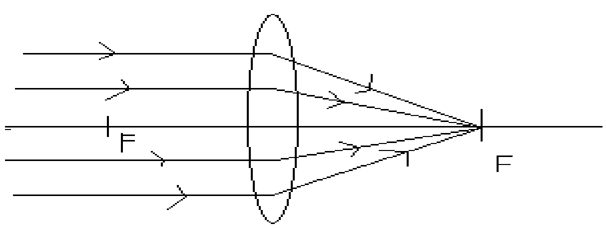
Convex lenses are thicker in the middle than at edges while concave lenses are thinker in the edges than at the middle.

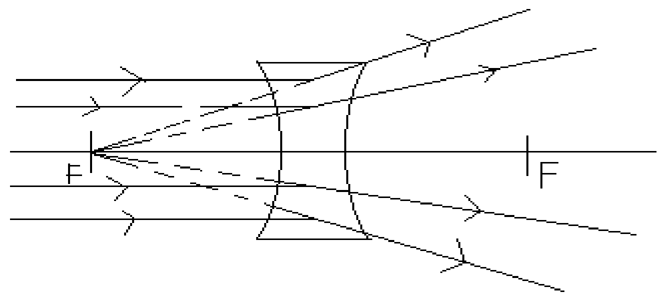


Centers of curvature

Centres of the sphere of which the spherical surfaces of the lens are part of principal axis. The line joining the centres of curvature on the surfaces of the lens.

Optical centre point on the centre of the lens and at the principle axis through which rays incident on the lens pass undeviated.





Principal focus of a thin lens is the point on the principal axis towards which all paraxial rays, parallel to the principal axis, converge in the case of the convex lens, or from which they appear to diverge in the case of the concave lens after refraction.

A lens has two principal foci one on each side and these are equidistant from the optical centre.

Focal length the distance between the optical center and principal focus.

A convex lens has *real foci* while a concave lens has *virtual foci*.

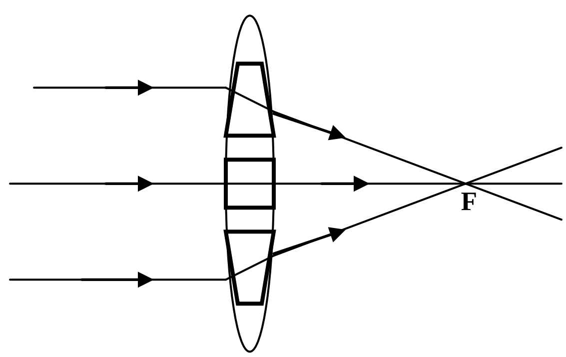
Power of the lens

The power, P, of the lens is the reciprocal of the focal length in metres

$$P = \frac{1}{f(m)}$$

Units of power *Dioptres (D)*

Explanation of action of the lens



A thin lens is regarded as made up of a large number of small angle prism whole angles increase from zero at the middle of the lense to a small value at the edge.

The deviation by a small angle prism is *d = (n-1)A* where n is the refractive index therefore, for light incident on a path of convex lens near the apex is deviated more than light incident near the middle part of the lens

For a concave lens the transparent prism point towards the centre of the lens

The one which is far away has a longer angle of deviation

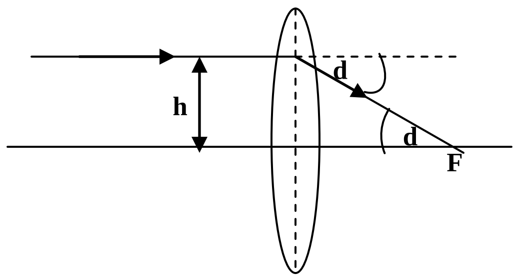
As you move to the centre, the angle *d* decreases.

Light incident on the point of lens near the middle lens is deviated lens than light incident on the point of lens further away from the middle of the concave lens.

Thin lens formula

Convex lens

Consider array of light incident on a lens close to its principal axis and parallel to it.

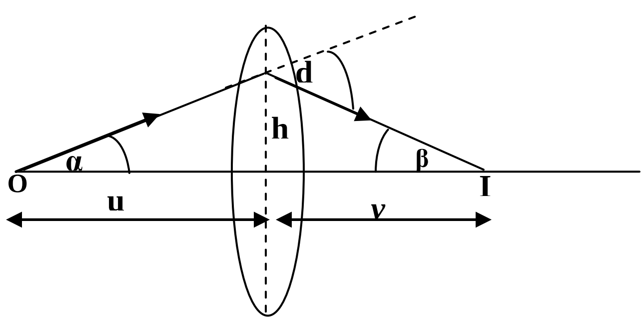


For paraxial rays and if d is small

$$d \approx \tan d = \frac{h}{f}$$

$$d = \frac{h}{f} \text{ ------------------------------ (i)}$$

Consider the formation of a real image by a real object. Also consider a ray from the object incident on the lens at a point distance h above the principal axis.



Since the deviation is independent of the angle of incidence of small angles $(d = (n-1)A)$, light is deviated through the same angle *d* .This follows because light is incident on the same point of the lens in both cases.

d = $\alpha + \beta$ (exterior < properties of an Δ) .

But for paraxial rays and small angles of α and β

$$\alpha \approx \tan \alpha = \frac{h}{u}$$

$$\beta \approx \tan \beta = \frac{h}{v}$$

$$d = \frac{h}{u} + \frac{h}{v}$$

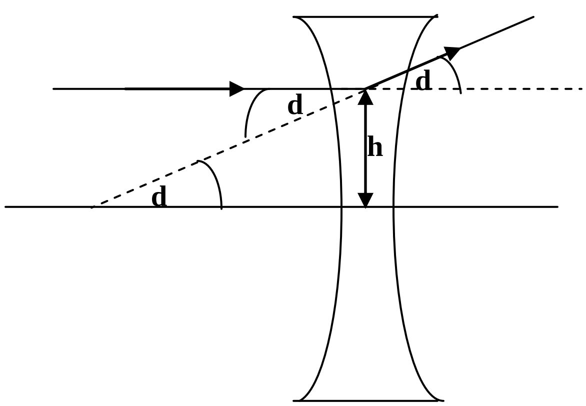
From (i) $d = \frac{h}{f}$

$$\frac{h}{f} = \frac{h}{u} + \frac{h}{v}$$

therefore $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

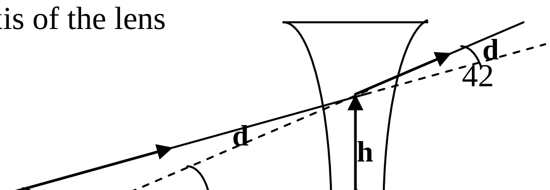
Concave lens

Consider array incident on a concave lens parallel and close to the principal axis.



Paraxial rays small angles $d \approx \tan d = \frac{h}{f}$

Consider a point object forming a virtual image. let the ray strike the lens at a point a distance h above the axis of the lens



For paraxial rays and for small angles of α and β

$$\alpha \approx \tan \alpha = \frac{h}{u}$$

$$\beta \approx \tan \beta = \frac{h}{v}$$

$$\alpha + d = \beta$$

$$d = \beta - \alpha$$

$$\frac{h}{f} = \frac{h}{v} - \frac{h}{u}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Considering real is positive, virtual negative sign convention, the principal focus and image of a concave lens are virtual hence f is negative and v is negative.

$$\frac{1}{-f} = \frac{1}{-v} - \frac{1}{u}$$

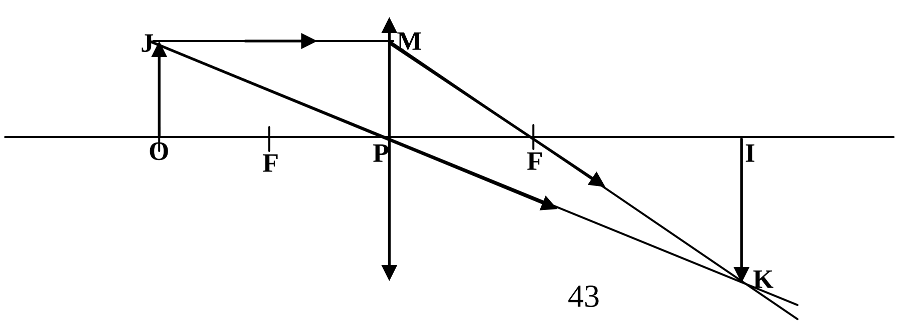
Therefore $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Exercise: *Following the example for mirrors, derive the lens formula using a finite object*

Linear and lateral magnification

$$magnification, M = \frac{imageheight}{ObjectHeight}$$

Consider formation of real image by a convex lens.



Δ OJP is similar to Δ IKP

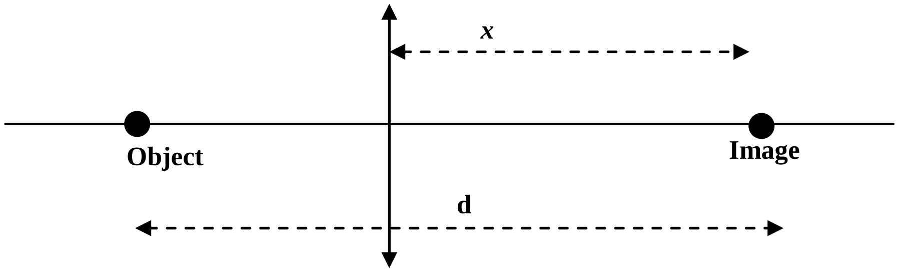
$$\frac{IK}{OJ} = \frac{PI}{OP} = \frac{v}{u} = \frac{image\ distance}{object\ distance}$$

But $\frac{IK}{OJ} = M$

Therefore M $=$ $\frac{\text{image distance}}{\text{Object distance}}$

Least distance between image distance and object distance

Least distance between an object and a real image formed by a convex lens.



Suppose the object and image are a distance d apart with the image a distance x beyond the lens

u = d – x , v = x

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{d - x}$$

$$dx - x^2 = df$$

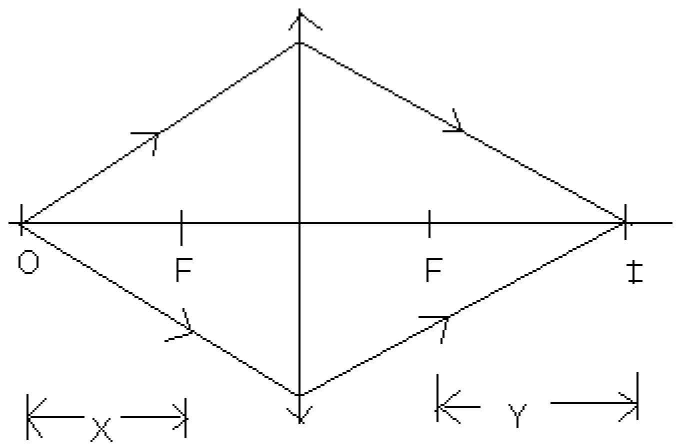
$$x^2 - dx + df = 0$$

For real values of x; $d^2 - 4df \geq 0$

$d \geq 4f$

The least distance between an object and the real image of the object formed by the convex lens is 4f.

Conjugate points



Suppose a convex lens forms an image of an object O at I ,if the object was placed at I, the lens would form the image of the object at O, then O and I are called conjugate points

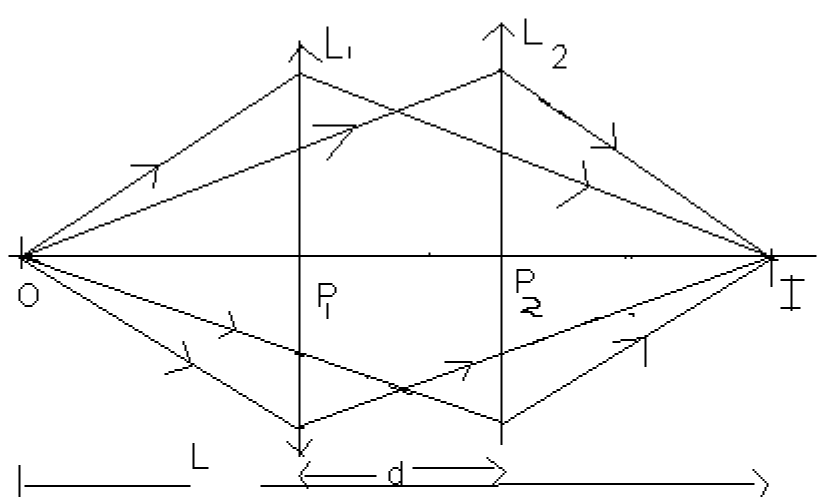
$u = x + f, v = y + f$

Use $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$\frac{1}{f} = \frac{1}{x+f} + \frac{1}{y+f}$

you get $f^2 = xy$ (Newton's equation)

Displacement of a lens when object and screen are fixed.



Object and image positions are fixed at O and I respectively.

The lens is moved in position L_1 when the image of O is produced at I, it's a gain displaced through a distance d until the image of O is a gain produced at I.

O and I are conjugate points.

$$OP_1 = P_2 I = \frac{L - d}{2}$$

Consider lens in position L_1

$$u = OP_1 = \frac{L - d}{2}$$

$$v = P_1 I = \frac{L - d}{2} + d = \frac{L + d}{2}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{2}{L + d} + \frac{2}{L - d}$$

$$4Lf = L^2 - d^2$$

Magnification when lens is at L_1

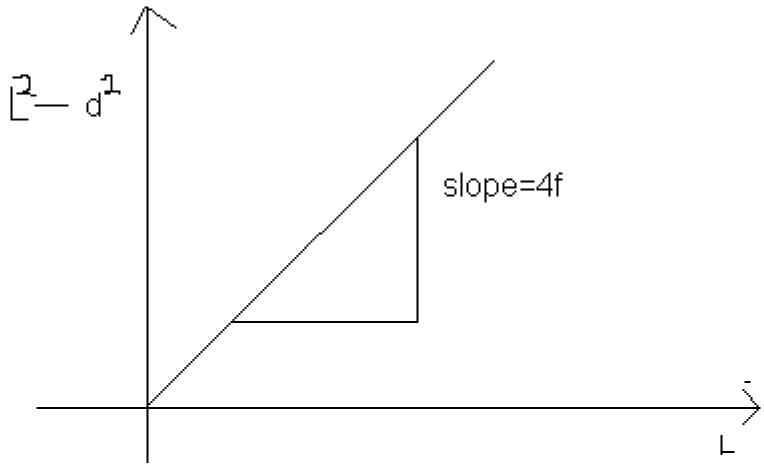
$$M_1 = \frac{v}{u} = \frac{\frac{L + d}{2}}{\frac{L - d}{2}} = \frac{L + d}{L - d}$$

Magnification when lens is at L_2

$$M_2 = \frac{\frac{L - d}{2}}{\frac{L + d}{2}} = \frac{L - d}{L + d}$$

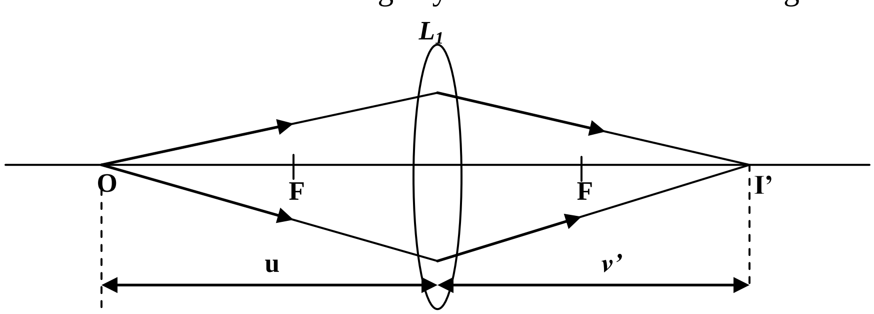
$$M_1 \times M_2 = \left(\frac{L + d}{L - d}\right) \times \left(\frac{L - d}{L + d}\right) = 1$$

In *determining the focal length of a convex lens*, different values of d are obtained for different object and screen distances l and a graph of L_2-d_2 is plotted against L



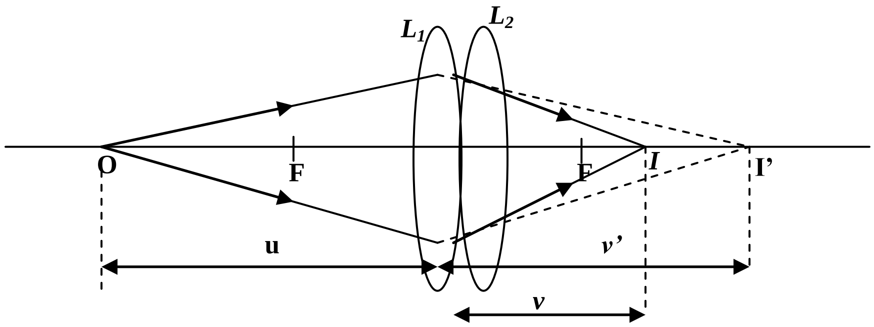
Combined focal length of two thin lenses in contact

Consider formation of real image by a convex lens of focal length f



$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v'} \quad \text{..........} \quad \text{..........} \quad \text{..........} \quad (1)$$

A second convex lens of focal length f_2 is now placed in contact with the first one.



I' acts as a virtual object for the second lens L_2, *v'* is negative

Action of lens L_2

$$\frac{1}{f_2} = \frac{1}{-v'} + \frac{1}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

equation (1) + (2)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Taking the combination to be equivalent to a single convex lens of focal length f, then- 1

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

from equation (3) and (4) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Example

A small object is placed at a distance of 60cm to the left of a diverging lens of focal length 30cm.A converging lens is then placed in contact with the diverging lens. If a real image is formed at a distance of 20 cm to the right of the combined lenses; find the focal length of the converging lens.

u = 60cm, v = 20cm

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{20} = \frac{1}{15}$$

$$f = 15\ cm$$

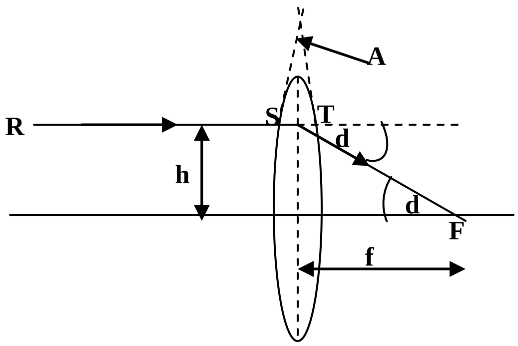
Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{15} = \frac{1}{f_1} + \frac{1}{-30}$$

$$f_1 = 10\ cm$$

Full thin lens formula

Consider the parallel rays RS incident on a convex lens



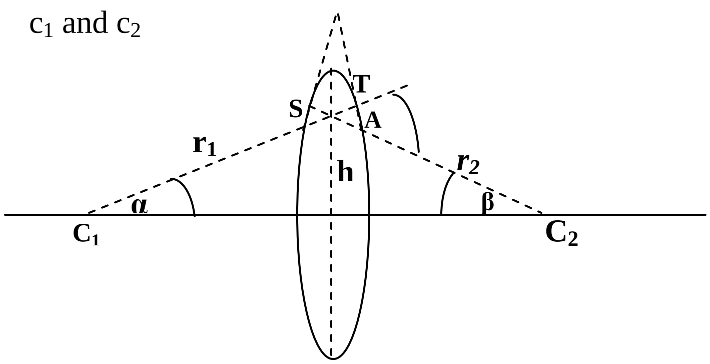
If A is the refracting angle of the small angle prism formed by tangents at S and T, deviation

$$d \approx \tan d = \frac{h}{f}$$

for small angle prism $d = (n-1)A$

$$\frac{h}{f} = (n-1)A \quad \text{--------------------} \ (1)$$

Consider the radii normal to the surfaces of the lens at Sand T passing through the centre of curture c_1 and c_2



$\alpha + \beta = A$ (using exterior < properties)

for α and β small angles in radians,

$$\alpha \approx \sin \alpha = \frac{h}{r_1}$$

$$\beta \approx \sin \beta = \frac{h}{r_2}$$

Hence $A = \frac{h}{r_1} + \frac{h}{r_2}$ ....(2)

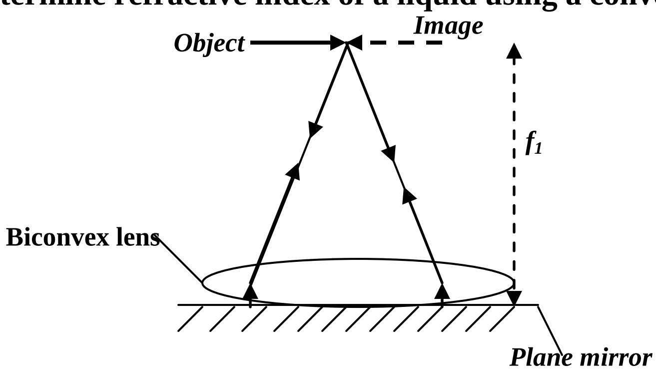
Substitute equation (1) into (2)

$$\frac{h}{f(n-1)} = \frac{h}{r_1} + \frac{h}{r_2}$$

$$\frac{1}{f(n-1)} = \frac{1}{r_1} + \frac{1}{r_2}$$

Hence $\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

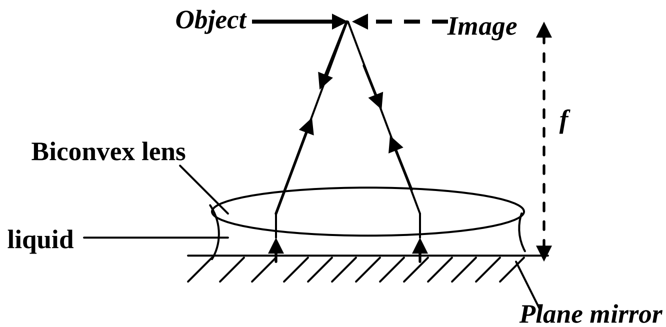
Determine refractive index of a liquid using a convex lens



A plane mirror is placed horizontally on a bench. A thin biconvex lens is placed on the mirror. The pin is clamped horizontally in a retort stand so that so that its pointed end lies along the axis a long the axis of the lens

The pin is moved along the axis until the position is reached where the image of the pin coincides with the pin itself. The distance f_1 of the pin from the lens is measured and is the focal length of the lens.

The lens is then removed from the plane mirror .A small quantity of the specimen liquid is placed on the mirror, the bi convex lens is then placed on top of the liquid.



The new position of the pin where it coincides with its image formed. The distance f of the pin from the lens is measured and is the focal length of the liquid –glass lens combination.

Using $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

The focal length f_2 of the liquid lens can be found.

Hence $\frac{1}{f_2} = (n_l - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Where $r_1 = r$ = radius curvature of glass lens

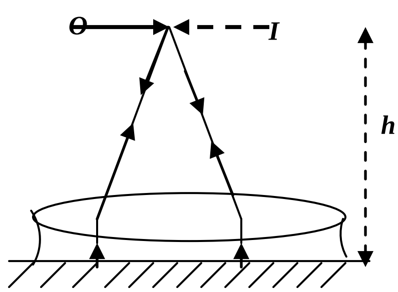
$r_2 = \infty$

$$\frac{1}{f_2} = (n_l - 1)\left(\frac{1}{r}\right)$$

$$n_l = 1 + \frac{r}{f_2}$$

Example

1. A converging lens is placed on top of a liquid of refractive index 1.4 and a glass slide as shown



using pin O position is found where O coincides with it images. If both surfaces of the lens has radii of curvature of 15cm and refractive index of the lens 1.5. Determine the distance h

$$n_l = 1 + \frac{r}{f_l}$$

$$1.4 = 1 + \frac{-15}{f_l}$$

$$\frac{1}{f_l} = -\frac{0.4}{15}$$

$$\frac{1}{f_g} = (n_h - 1)\left(\frac{1}{r}\right)$$

$$\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{15}\right) = \frac{0.5}{15}$$

hence $\frac{1}{h} = \frac{1}{f_l} + \frac{1}{f_g} = \frac{0.4}{-15} + \frac{0.5}{15} = \frac{1}{25}$

$$h = 25\ cm$$

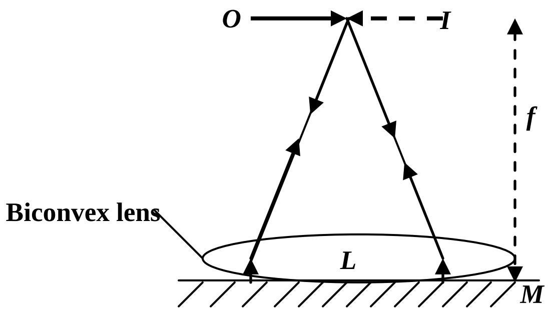
Exercise

1. A biconcave lens of radius of curvature 24cm is placed on a liquid film on a plane mirror. A pin clamped horizontally above the lens coincides with image at a distance of 40cm above the lens. If the refractive index of the liquid is 1.4 ,what is the refractive index of the material of the lens.(1.5)
2. A thin biconvex les is placed on a plane mirror. A pin is clamped above the lens so at it apex lies on the principal axial of the lens. The position of the pin is adjusted until the pin coincides with its image at a distance of 15cm from the mirror. When a thin layer of water of refractive index 1.33 is placed between the mirror ,the pin coincides with its image at a point 22.5cm from the mirror. When water is replaced by paraffin, the pin coincides with the image at a distance of 27.5cm from the mirror. Calculate the refractive index of paraffin.(1.45)

Determining focal length of the converging lens(convex lens)

1. plane mirror method

A plane mirror M is placed on a table and the lens L is placed on the mirror. A pin O is then moved a long the axis of the lens until its image I coincides with the object O, when both are viewed from above and there is no parallax.

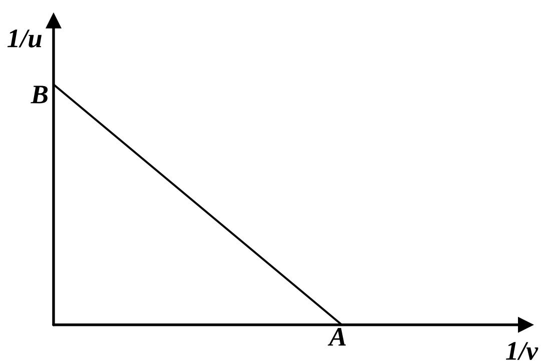


The distance from the pin O to the lens is the focal length ,f, of the lens and it is thus measured. Rays from O pass through the lens are reflected from the mirror M and then pass through the lens a gain to form an image .*When O and I coincides the rays from O incident on the mirror must have returned a long their incident path after reflection from the mirror. This happens if the rays are incident normally on M, the rays entering the lens after reflectin are parallel and hence the point at which they converge must be the focus.*

2. Lens formula method

In this method,5 or 6 values of ***u*** and ***v*** are obtained by using an illuminated object and a screen, or by using two pins of no parallax. The focal length can be calculated from the equation $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, and the average of the values obtained

Alternatively the values of $\frac{1}{u}$ can be plotted against $\frac{1}{v}$ and a straight line drawn through the points



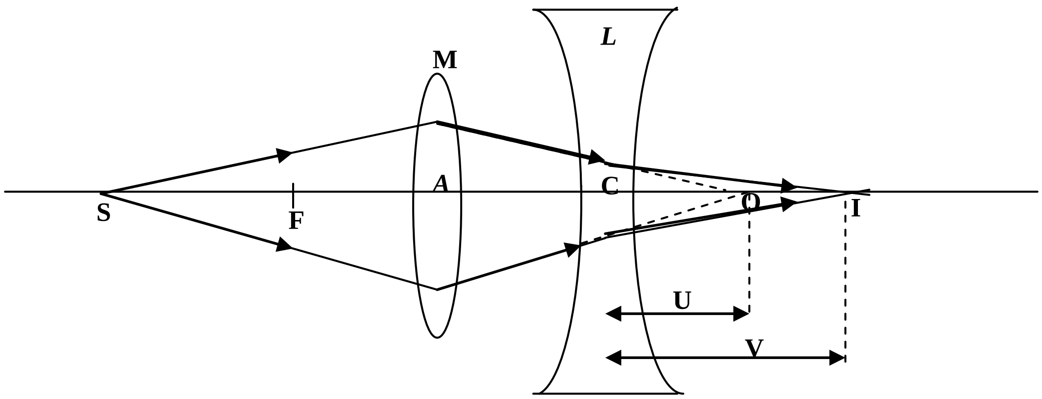
The intercepts A and B are read from the graph and focal length, *f* is obtained from the equation

$$f = \frac{2}{A + B}$$

Diverging lens

1. Converging lens method

by itself, a diverging lens always form a virtual image of a real object. A real image may be obtained, however, if a virtual object is used ,and a converging lens can provide such an object. An object S is placed at a distance from M eater than its focal length, so that a beam converging to a point O is thus a virtual object for the diverging lens L and a real image I can be obtained.



I is further a way from L and O ,since the concave lens makes the incident beam unit diverge more. The distance (image),v, from the diverging lens is CI and can be measured; v is positive in sign as I is real. The object distance U, from this lens is LO= AO – AC, and AC can be measured. The length AO is obtained by removing the lens L leaving the converging lens and noting the position of the real image now formed by O by the lens M. This U = CO can be found; its negative ,since O is virtual for the concave lens so, from $\frac{1}{f} = \frac{1}{U} + \frac{1}{V}$,focal length *f* is calculated.

2) Concave mirror method

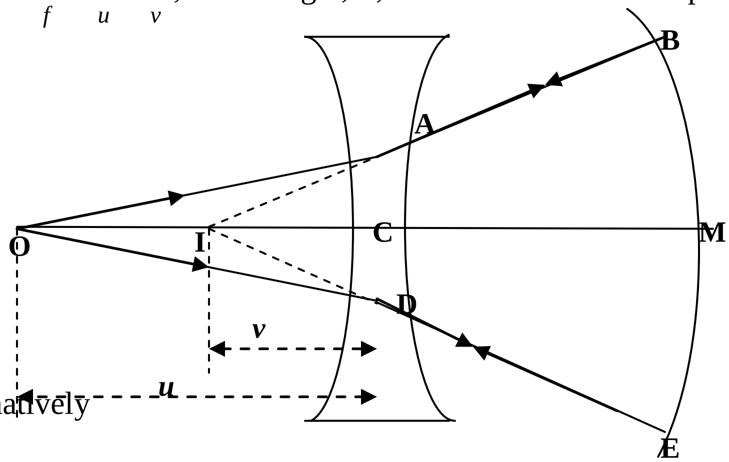
A real object is placed in front of the diverging lens and the position of the virtual image is located with the aid of a concave mirror. An object O is placed in front of the lens L, and a concave mirror M is placed behind the lens so that a diverging beam is incident on it with L and M in the same position, the object O is moved until an image is obtained coincide it in position, besides O. The distance CO,CM are then measured. As the object and image are coincident at O, the rays may be incident normally on the mirror M .The rays BA, ED thus pass through the centre of curvature of M, and this is also the position of the virtual image I. The image distance, v, from the lens v = IC = IM – CM = r- CM, where r is the radius of curvature of the mirror, CM is measured.

r is determined separately using a concave mirror and placing the object at the centre of curvature

u = r = v

The object distance u, from the lens = OC

Using $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, focal length, f, can be calculated. *v* is positive as I is virtual

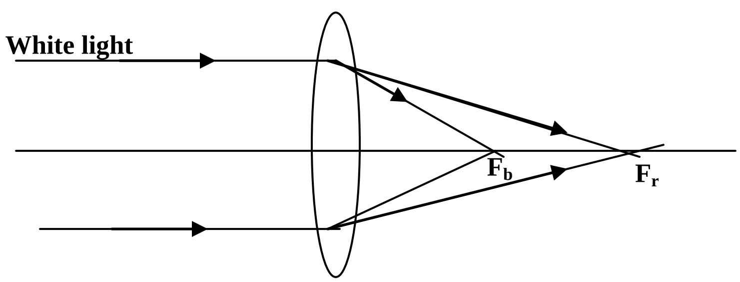


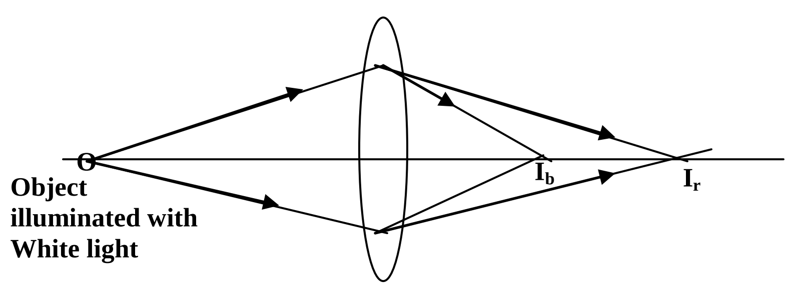
Alternatively

Defects of image formed by lenses

1. **Chromatic aberration**

When white light is incident on a lens, the different colour components are deviated by different a mounts. Red light is deviated less than the blue light. As a result, images corresponding to the different colours are formed in different locations along the principal axis of the lens.

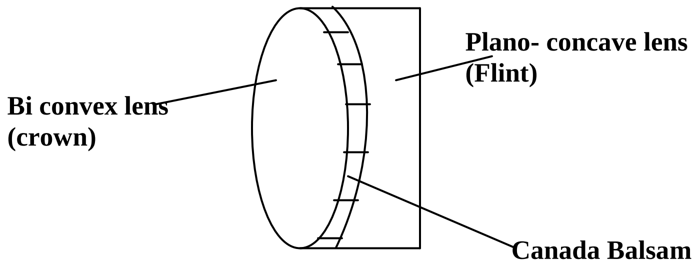




This causes the image of an object illuminated by white light to be blurred with colored edges.

Minimizing chromatic aberration

Chromatic aberration can be reduced by using Achromatic doublet. This consists of a bi-convex lens just in contact with a Plano concave lens different refractive indices, using Canada balsam.



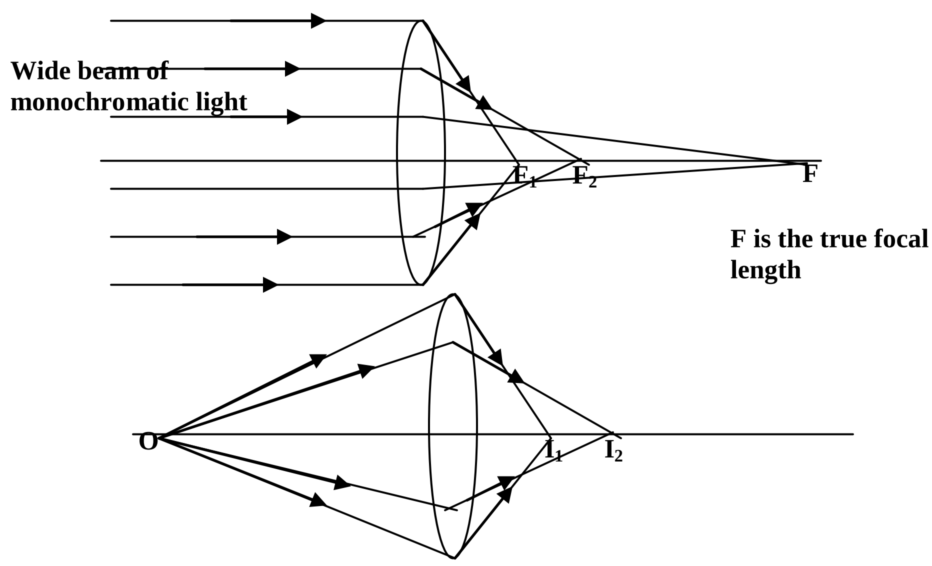
2. Spherical aberration.

This happens in both mirrors and lenses**.**

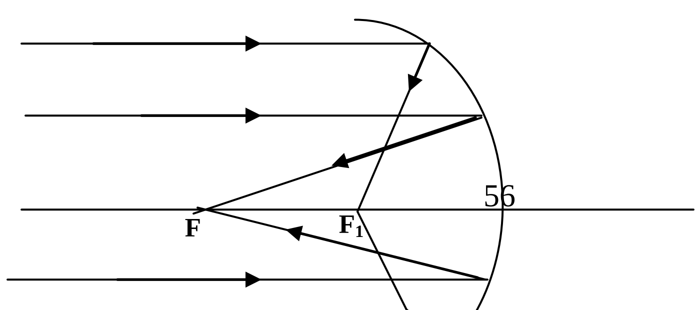
Consider a wide beam of monochromatic light incident on a wide aperture converging lens.

Marginal rays (far a way from the principal focus) converge near the lens than the central rays (paraxial rays).

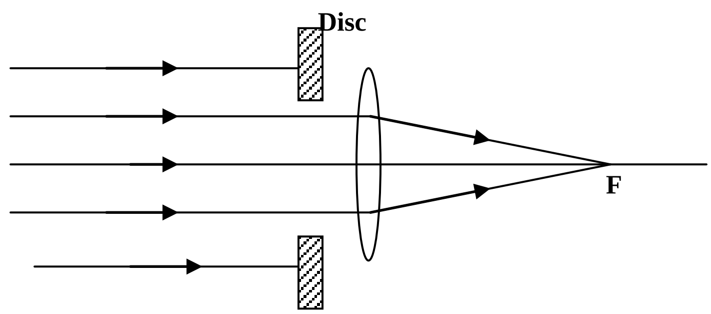
Consequently the image formed is blurred. This defect of the image is called spherical aberration.



This defect also occurs with wide aperture spherical mirror.

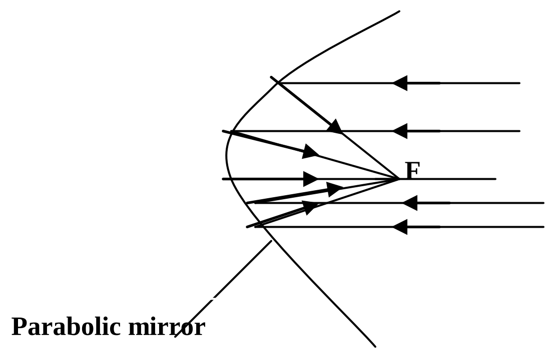


Spherical aberration can be minimized using a stopper i.e. using an opaque disc with a central hole.



The disc will stop marginal rays from reaching the lens. The disadvantage with this method is light intensity is cut down and so the brightness of the image is reduced.

In mirrors, spherical aberration can be minimized by using a parabolic mirror instead of a concave mirror.



This brings all rays parallel to principal axis to converge at the same focus.

Exercise

1. An illuminated object is placed on the 0cm mark of an optical bench. A converging lens of focal length 15cm is placed at the 22.5cm mark. A diverging lens of focal length 30cm and a plane mirror are placed at the 37.5cm and 77.5cm marks respectively

Find the position of the final image.(at 0cm mark). Illustrate your answer with a ray diagram

2. A converging lens of focal length 1.5cm is placed 29.0cm in front of another converging lens of 6.25cm.An object of height 0.1cm is placed 1.6cm a way from the first lens on the side remote

from the second lens at right angles to the principal axis of the final image by the system. Determine the position and size and final image of the object.

3. An equi-convex lens A is made of glass of refractive index 1.5 and has a power of 5.0$radm^{-1}$. It is combined in contact with a lens B to produce a combination whose power is 1.0$radm^{-1}$. the surfaces in contact fits exactly. The refractive index of the glass in lens B is 1.6. What are the radii of the four surfaces? Draw a diagram to illustrate your.

4. A lens forms the image of a distant object on a screen 30cm away. Where should a second lens of focal length 30cm be placed so that the screen has to be moved 8cm towards the first lens for the new image to be in focus.

5. A convex lens of focal length 20cm, forms an image on a screen placed 40cm beyond the lens . A concave lens of focal length 40cm is then placed between a convex lens and a screen a distance of 20cm from the convex lens.

(i) Where must the screen be placed in order to receive the new image?

(ii) What is the magnification produced by the lens system?

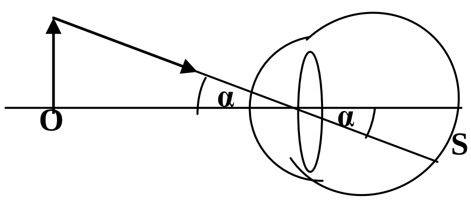
Optical instruments.

Least distance of distinct vision, D $\approx$ 25cm.

It's the shortest distance of the eye from the object at which the eye can clearly.

Virtual angle

Angle subtended at the eye by the object or it the angle subtended at the eye by the image when instrument is being used.



α -visual angle

arc length S = size of the image.

S = αa

Where a is the distance between the eye lens and the retina.

$\therefore$ Size of the image formed at the retina is directly proportional to the visual angle.

Explain why the further vertical pole in line with others of equal height looks shorter.

The image size depends on the visual angle. The farthest pole subtends a small visual angle Image size is smaller

Angular magnification (magnifying power).

It is the ratio of the angle subtended at the eye by the image when instrument is being used to the angle subtended at an unaided eye by the object.

i.e. angular magnification, $m = \frac{\alpha'}{\alpha}$

Where α' is angle subtended at the eye by the image when instrument is used and

α is the angle subtended at an unaided eye by the object.

Microscopes

These are used to view near objects

Angular magnification of microscopes $= \frac{\alpha'}{\alpha}$

α is the angle subtended at the eye object at the near point when microscope is not used.

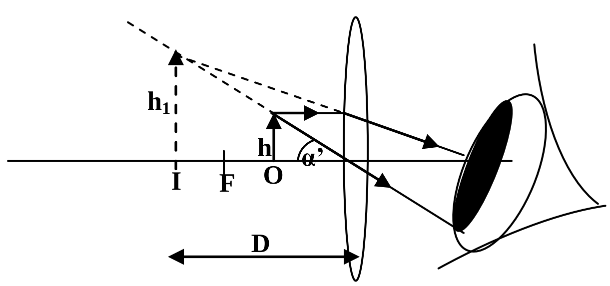
α' is the angle subtended at the eye by image when microscope is used.

In normal adjustment or use, the microscope forms the image at the near point.

Simple microscope / magnifying glass

This consists of a single convex lens with the distance between the object and the lens than or equal to the focal length of the lens.

Simple microscope with image at near point

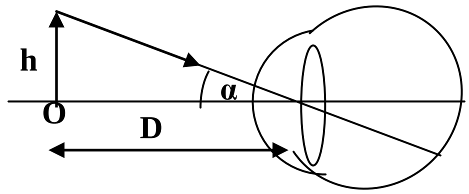


$m = \frac{\alpha'}{\alpha}$

For small angles in radians

$\alpha' \approx \tan \alpha' = \frac{h_1}{D}$

If α is the angle subtended at the eye by the object at the near point



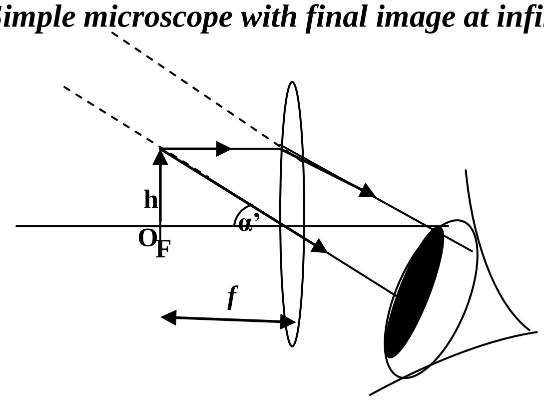
$$\alpha \approx \tan \alpha = \frac{h}{D}$$

hence $$m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h_1}{D}\right)}{\left(\frac{h}{D}\right)} = \frac{h_1}{h}$$

But $\frac{h_1}{h}$ is the linear magnification. Hence $\frac{h_1}{h} = \frac{D}{f} - 1$

There fore angular magnification, $m = \frac{D}{f} - 1$

Simple microscope with final image at infinity



Angular magnification, $m = \frac{\alpha'}{\alpha}$

$$\alpha' \approx \tan \alpha' = \frac{h}{f}$$

$$\alpha \approx \tan \alpha = \frac{h}{D}$$

Hence angular magnification, $m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h}{f}\right)}{\left(\frac{h}{D}\right)} = \frac{D}{f}$

Note: (i) Angular magnification is higher when a simple microscope forms the image at the near point.

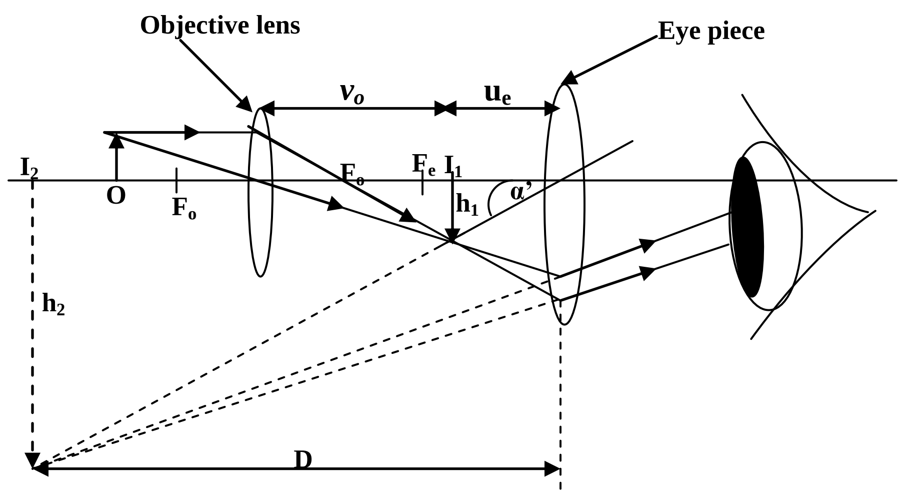
(ii) For higher magnification, use lenses of short focal length.

Compound microscopes

f can not be decreased indefinitely to achieve high angular magnification. This requires the use of two converging lenses, namely the objective (which is near the object) and the eye piece, near the eye.

Compound microscope in normal adjustment.

In normal adjustment, a compound microscope forms the image at the least distance of distinct vision from the eye i.e. image at near point



The objective lens forms a real image I_1, of the object O .I_1 is formed at a point nearer the eye piece than the principal focus f_e of the eye piece.

The eye piece acts as a magnifying glass. It forms a virtual image I_2 of I_1. The observer's eyes should be taken to be close to the eye piece so that α' is the angle subtended at the eye by the final image I_2.

For small angles in radians,

$$\alpha' \approx \tan \alpha' = \frac{h_2}{D}$$

But $\alpha \approx \tan \alpha = \dfrac{h}{D}$

$$m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h_2}{D}\right)}{\left(\frac{h}{D}\right)} = \frac{h_2}{h}$$

$$m = \left(\frac{h_1}{h}\right) \times \left(\frac{h_2}{h_1}\right) = \left(\frac{v_o}{f_o} - 1\right) \times \left(\frac{D}{f_e} - 1\right)$$

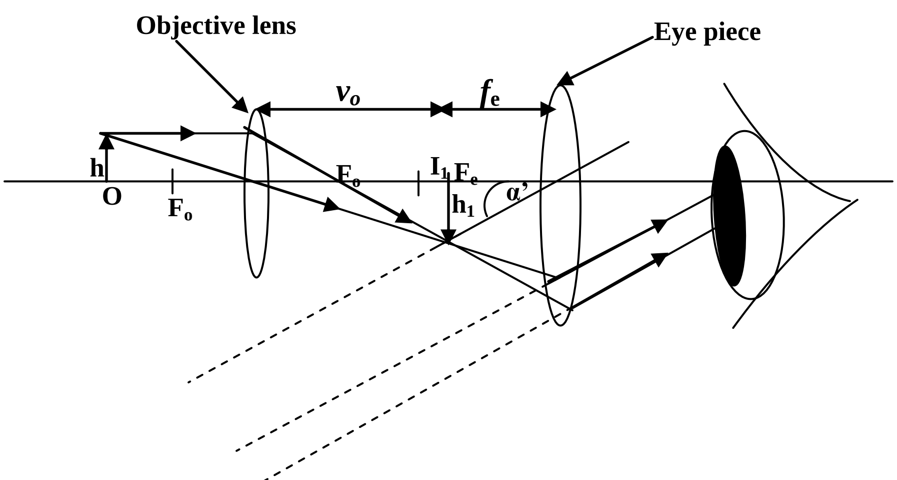
where,

$$\frac{h_2}{h_1} = \left(\frac{D}{f_e} - 1\right)$$

$$\frac{h_1}{h} = \left(\frac{v_o}{f_o} - 1\right)$$

Note : For higher angular magnification, both the eye piece and the objective should have short focal lengths.

Compound microscope with final image at infinity



The separation of the object and the eye piece is such that the object forms an image of the object at the principle focus Fe of the eye piece, hence the eye piece focuses the final image at infinity. The angle α' subtended by the final image by the eye piece is

$$\alpha' \approx \tan \alpha' = \frac{h_1}{f_e}$$

but $\alpha \approx \tan \alpha = \frac{h}{D}$

hence $m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h_1}{f_e}\right)}{\left(\frac{h}{D}\right)} = \left(\frac{D}{f_e}\right) \times \left(\frac{h_1}{h}\right)$

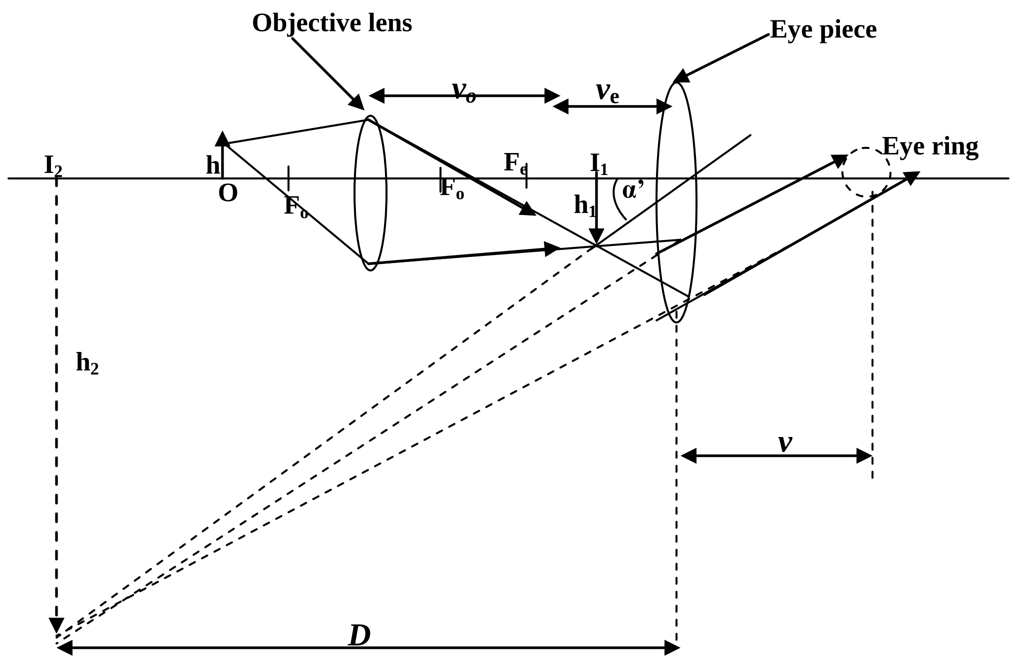
But $\frac{h_1}{h} = \left(\frac{v_o}{f_o} - 1\right)$

$$m = \left(\frac{D}{f_e}\right) \times \left(\frac{v_o}{f_o} - 1\right)$$

Eye ring/ Exit pupil

For a given objective lens, the best position of the observer's eye is that where it collects all rays coming from the objective lens. This position is called the *eye ring or exit pupil*.

Hence eye ring is the position of the image of the objective formed by the eye piece.



Note: when determining the eye ring, the separation is taken as the object distance and focal length of the eye piece is used in calculations. Hence from the above

$u = v_o + u_e, \ f = f_e$

Then use

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f_e} = \frac{1}{u_e + v_o} + \frac{1}{v}$$

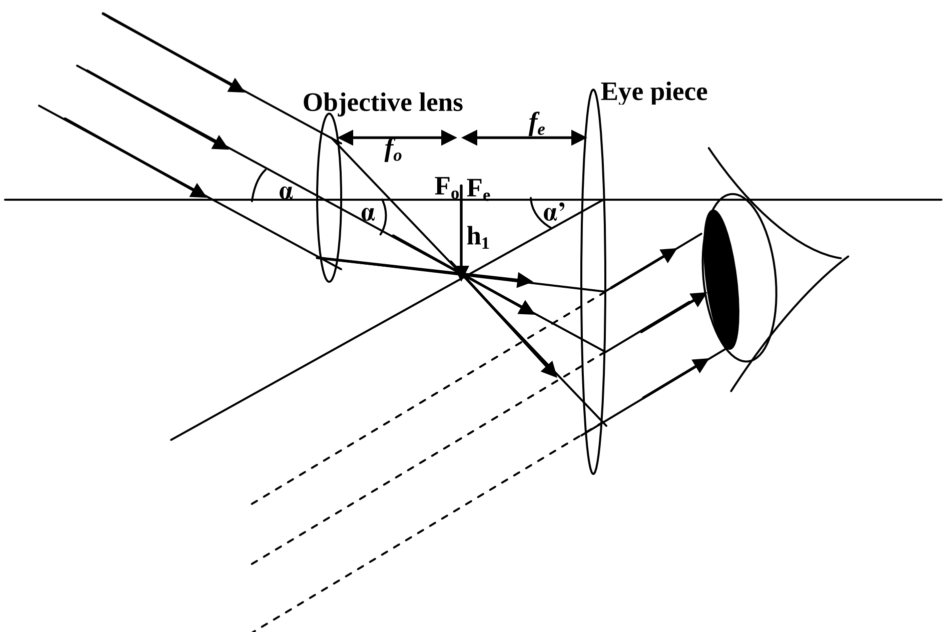
hence ***v,*** which is the eye ring can be obtained.

Telescopes

Telescopes are used to view distant objects. The angular magnification of a telescope is the ratio of the angle subtended by the final image at the aided eye to the angle subtended by the object at the un aided eye. In normal adjustment, the final image is at infinity.

Astronomical telescope in normal adjustment

This consists of two converging lenses i.e. a long focal length objective lens and a short focal length eyepiece.



In normal adjustment, the image of the distant object formed by the objective lens lies in the focal plane of both the objective and the eye piece. Hence separation between the lenses $= f_o + f_e$

Angular magnification $m = \frac{\alpha'}{\alpha}$

$$\alpha' \approx \tan\alpha' = \frac{h_1}{f_e}, \quad \alpha \approx \tan\alpha = \frac{h_1}{f_o}$$

$$m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h_1}{f_e}\right)}{\left(\frac{h_1}{f_o}\right)} = \frac{f_o}{f_e}$$

The eye ring, and relation to angular magnification

$u = f_o + f_e, \quad f = f_e$

using

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

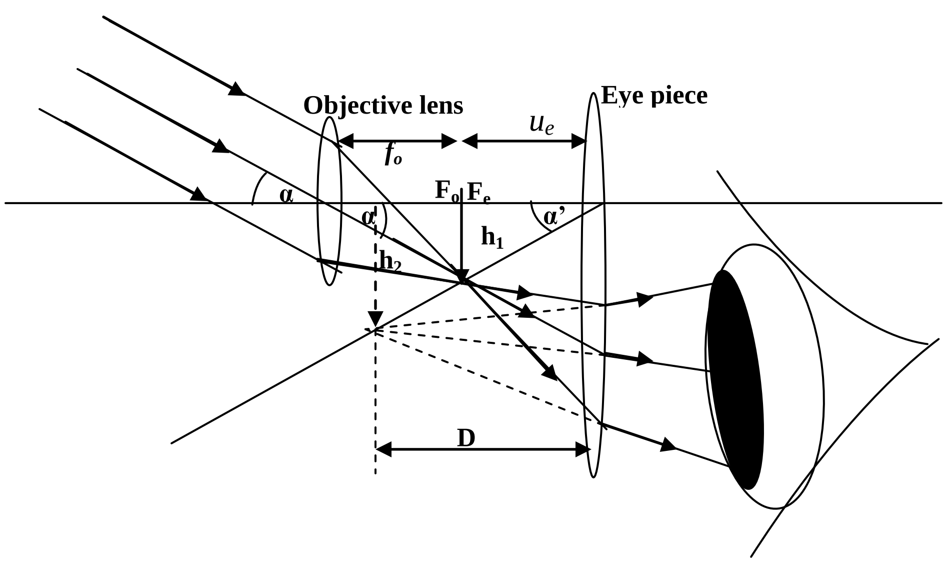
$$\frac{1}{f_e} = \frac{1}{f_e + f_o} + \frac{1}{v}$$

hence $v = \frac{f_e}{f_o}(f_o + f_e)$

diameter of eye ring ÷ diameter of objective = $\frac{v}{u} = \left(\frac{f_e}{f_o}(f_o + f_e)\right) \div (f_o + f_e) = \frac{f_e}{f_o}$

hence angular magnification $m = \frac{f_o}{f_e} = \frac{ObjectiveD \quad iameter}{eyeringdia \quad meter}$

Astronomical telescope with image formed at near point



The intermediate image should be formed in front of the focal point of the eye piece.

Angular magnification $m = \frac{\alpha'}{\alpha}$

But $\alpha' \approx \tan \alpha' = \frac{h_2}{D}$

$\alpha \approx \tan \alpha = \frac{h_1}{f_o}$

$$m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h_2}{D}\right)}{\left(\frac{h_1}{f_o}\right)} = \frac{f_o}{D} \times \frac{h_2}{h_1}$$

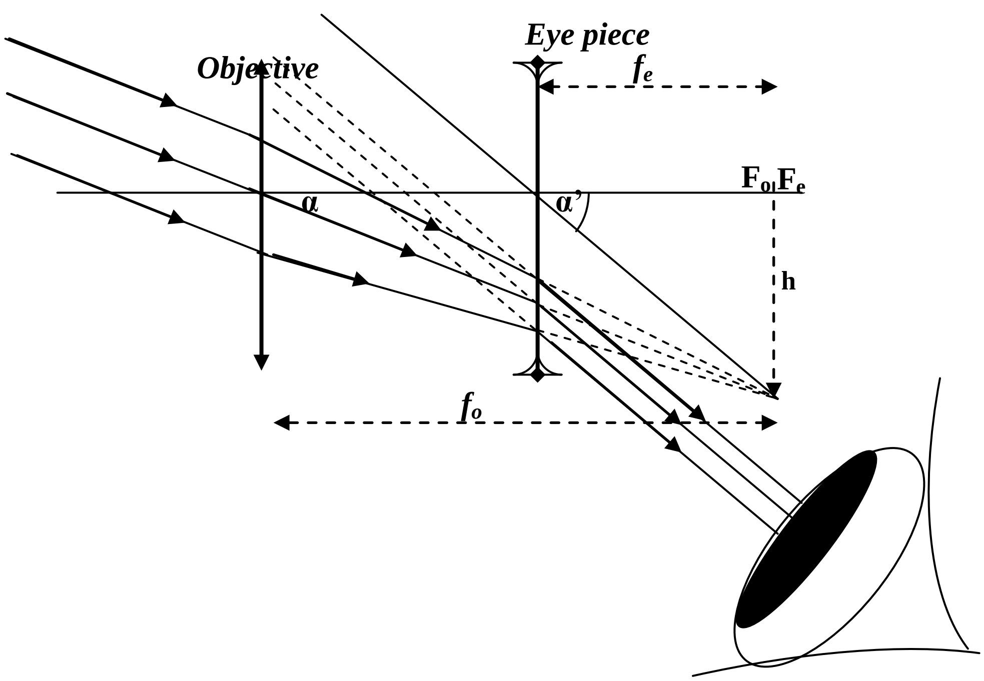
But

$$\frac{h_2}{h_1} = \left(\frac{D}{f_e} - 1\right)$$

Hence $m = \frac{f_o}{D} \times \left(\frac{D}{f_e} - 1\right) = \frac{f_o}{f_e} \times \left(1 - \frac{f_e}{D}\right)$

Galilean telescope

A Galilean telescope consists of a convex objective lens and a concave eye piece lens. The objective has a longer focal length than the eye piece.



$$\alpha' \approx \tan\alpha' = \frac{h}{f_e}$$

$$\alpha \approx \tan\alpha = \frac{h}{f_o}$$

$$m = \frac{\alpha'}{\alpha} = \frac{\left(\frac{h}{f_e}\right)}{\left(\frac{h}{f_o}\right)} = \frac{f_o}{f_e}$$

Exercise: Draw a Galilean telescope with the final image at the near point and show that the angular magnification is the same as that of an astronomical telescope.

Advantages of Galilean Telescope:

(i) It is shorter than astronomical telescope when in normal adjustment, hence it used for *opera glasses*

(ii) The final image is upright or erect.

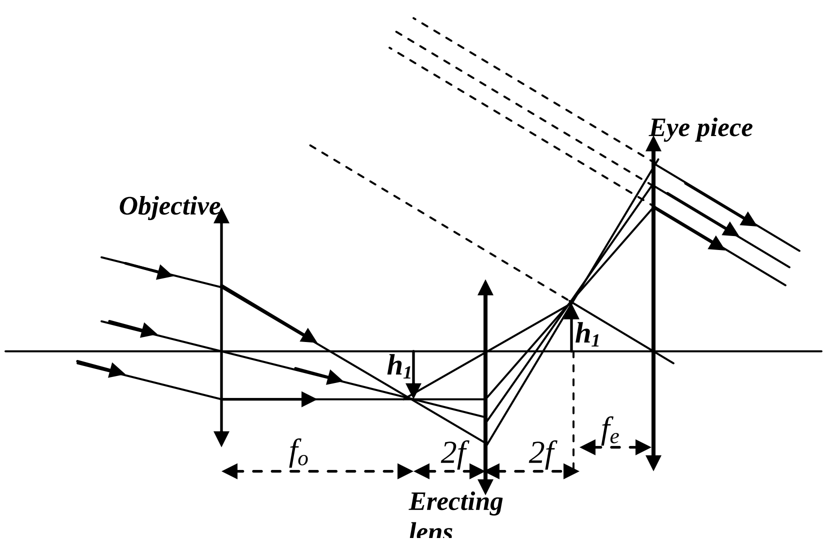
Disadvantages of Galilean Telescope:

(i) it has a virtual eye ring.

(ii) it has a limited field of view.

Terrestrial telescope

It is a refracting telescope with an intermediate erecting lens between the objective and the eye piece.



Advantage of terrestrial telescope

Produces upright images

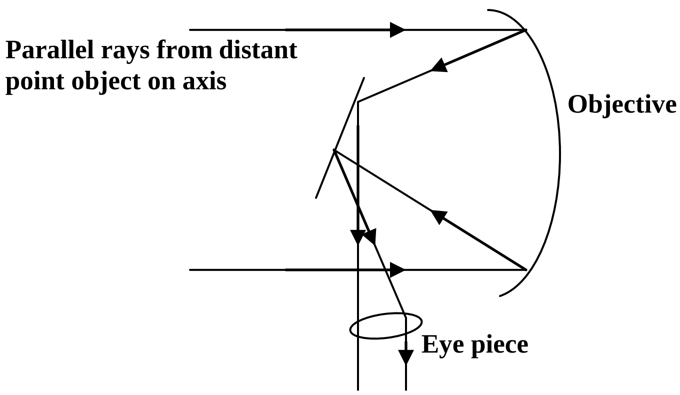
Disadvantages of terrestrial telescope

(i) erecting lens reduce the intensity of light emerging from the eye piece, as light is reflected at the lens surfaces.

(ii) Separation($4f + f_o + f_e$) of lenses is longer than any other telescope.

Newton's reflecting telescope

It consists of a concave mirror of long focal length as the objective instead of a convex lens, a plane mirror and a convex eye piece.



The objective mirror reflects rays from the object onto the plane mirror which reflects the rays onto the eye piece lens through which the image is observed. In normal adjustment, the final image is inverted and angular magnification, $m = \frac{f_o}{f_e}$

Advantages of reflecting telescope over refracting telescope

(i) There is no chromatic aberration since no refraction occurs at the objective which is a mirror.

(ii) Spherical aberration can be reduced easily by using a parabolic mirror as the objective.

(iii) It is much cheaper and easier to make a mirror than a lens since one surface requires to be grounded.

(iv) Images are brighter because it is easy to make mirrors of large aperture which collects a lot of light compared to making a lens with large aperture

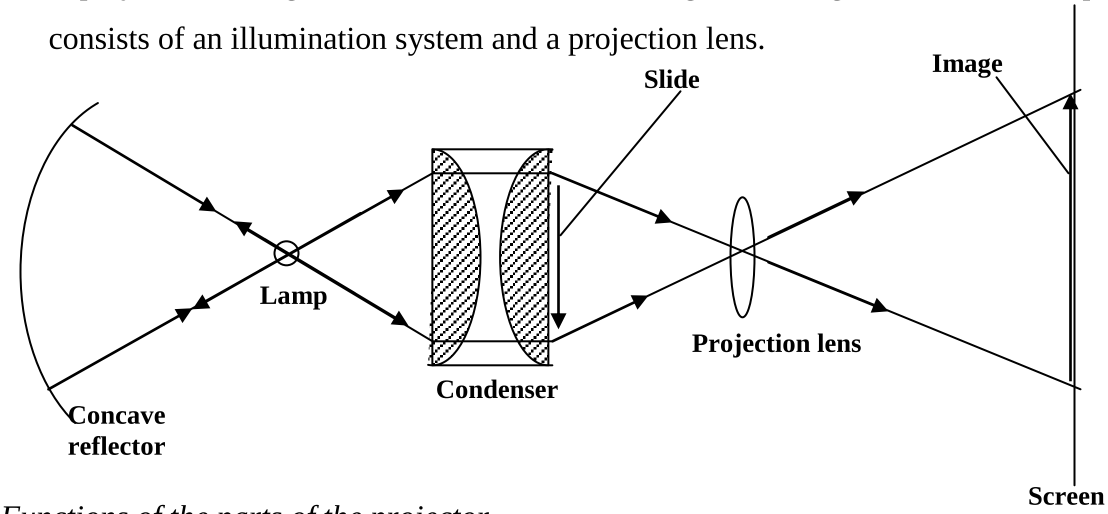
(v) Resolving power is also greater (i.e. seeing different images as separate)

Disadvantage of reflecting telescope over refracting telescope

(i) they tarnish easily and absorb light resulting in dull images.

Projector Lantern

A projector is designed to throw on a screen a magnified image of a film or transparency. It consists of an illumination system and a projection lens.



Functions of the parts of the projector

(i) the concave reflector concentrates light from the source on to the condenser.

(ii) The condenser uniformly illuminates the object with light from the source. Or it concentrates light toward the object.

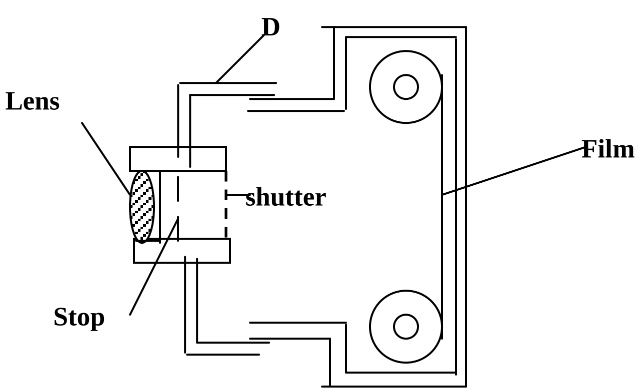
(iii) The slide bears the object to be projected to the screen

(iv) Projection lens focuses and magnifies an upright image on the screen

(v) Screen is where the image is viewed.

The lens camera

A camera consists of a lens system, a light sensitive film at the back and a focusing arrangement D. The latter is used to adjust the distance of the lens for proper focusing.



The lens in the camera is achromatic doublet, hence it is free from chromatic aberration.

The stop or aperture is provided so that the light is incident centrally on the lens, thus diminishing distortation. -*